

Teaching Modelling and Systemic Change

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Malcolm Swan

Shell Centre
Centre for Research in Mathematics Education
University of Nottingham

ICTMA 2015

The Shell Centre 'Brief'

Agreed with the University in 1976:

**To work to improve the
teaching and learning of mathematics
regionally, nationally and internationally**

**Impact remains a key focus of Shell Centre projects
within a broad range of CRME research**

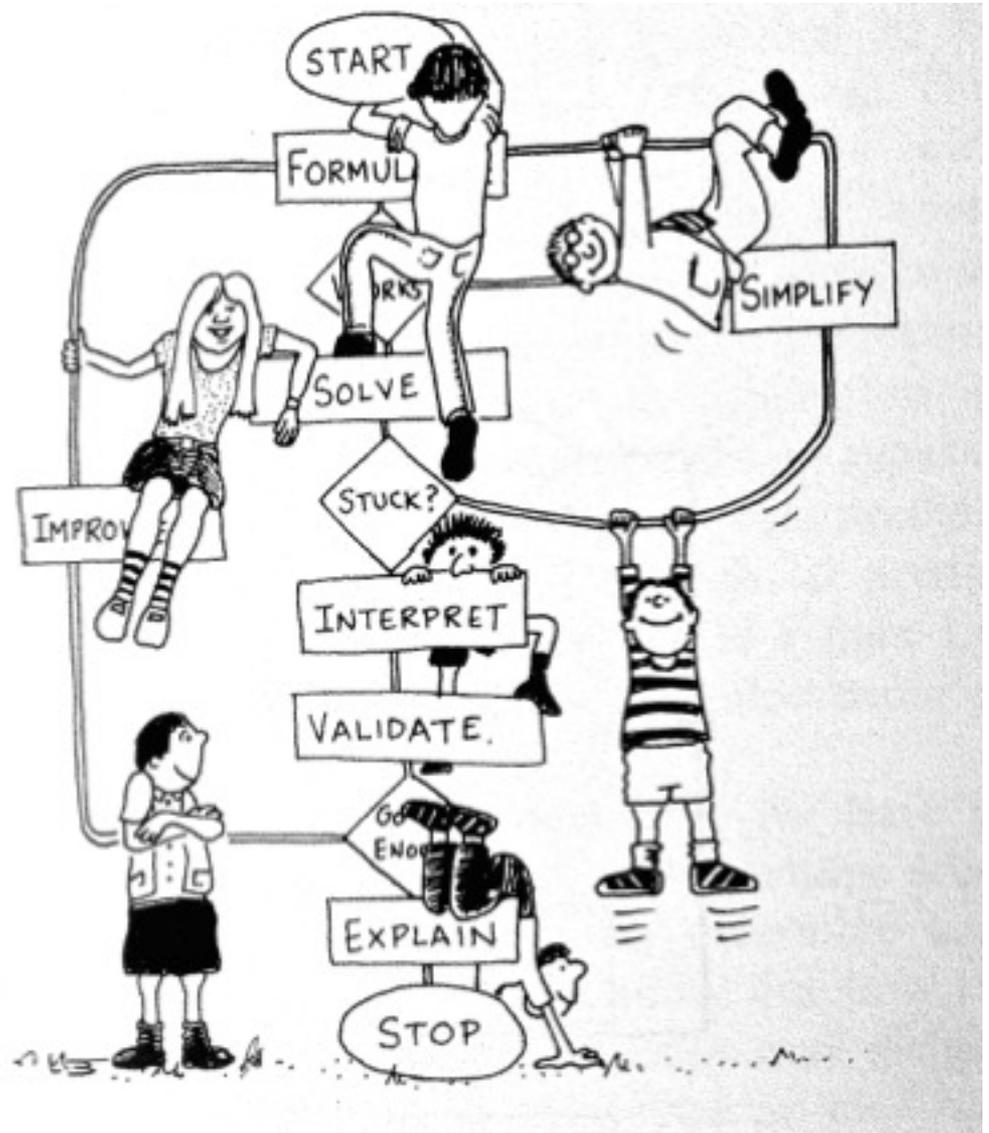
Aiming to transform practice through design research:

- Analysing existing situations
- Designing new processes, products and experiences for teachers and learners
- Articulating values and principles that underpin these “designs”
- Analysing “designs in action”
- Revising and refining theories and designs in the light of these experiences
- “Scaling up” designs for use by others.

Projects that have involved Modelling

- 1981** **The Real World and Mathematics**
- 1981 - 1986** **Testing Strategic Skills**
Task exemplars, Boxes, WYTIWYG, Alignment, Gradual change
- 1987 - 1989** **Numeracy through Problem Solving**
Modelling in maths, materials-directed project work, group investigations, exams on projects, controlled transfer distance
- 1985 - 1988** **Extended tasks for GCSE Maths**
Materials supporting 'coursework'/portfolios
- 1999 - 2005** **World Class Arena**
Test of PS across STEM, Expert tasks ~ high strategic/low technical demand, computer-based tasks, microworlds
- 2007 - 2014** **Bowland Maths**
Investigative video/software driven microworlds, PD package
- 2011 - 2013** **Promoting Inquiry in Mathematics and Science**
Professional Development for Inquiry-based Learning
- 2009 - 2014** **Mathematics Assessment Project**
Supporting formative assessment through materials
- 2014 - 2015** **Lessons for Mathematical Problem Solving (LeMaPS)**
Using Lesson Study for Professional Development in Modelling





Where to find the research products

<http://map.mathshell.org>

<http://mathshell.org>



The University of
Nottingham

UNITED KINGDOM • CHINA • MALAYSIA

Centre for Research into
Mathematical Education



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Downloads & Licensing

Math Assessment Project

Formative Assessment For the
Common Core State Standards

**BOWLAND
MATHS**

Resources for KS3

INTERNATIONAL
SOCIETY FOR
DESIGN AND
DEVELOPMENT IN
EDUCATION

Primas

PROMOTING INQUIRY
IN MATHEMATICS AND SCIENCE
EDUCATION ACROSS EUROPE

Toolkit for **Change**

Resources for Leaders in Mathematics
Education

ED EDUCATIONAL
DESIGNER

Journal of the International Society for
Design and Development in Education



World Class Arena
challenging minds, recognising talent™

Tests of Problem Solving in
Mathematics Science and Technology

Our Current Team



Overview

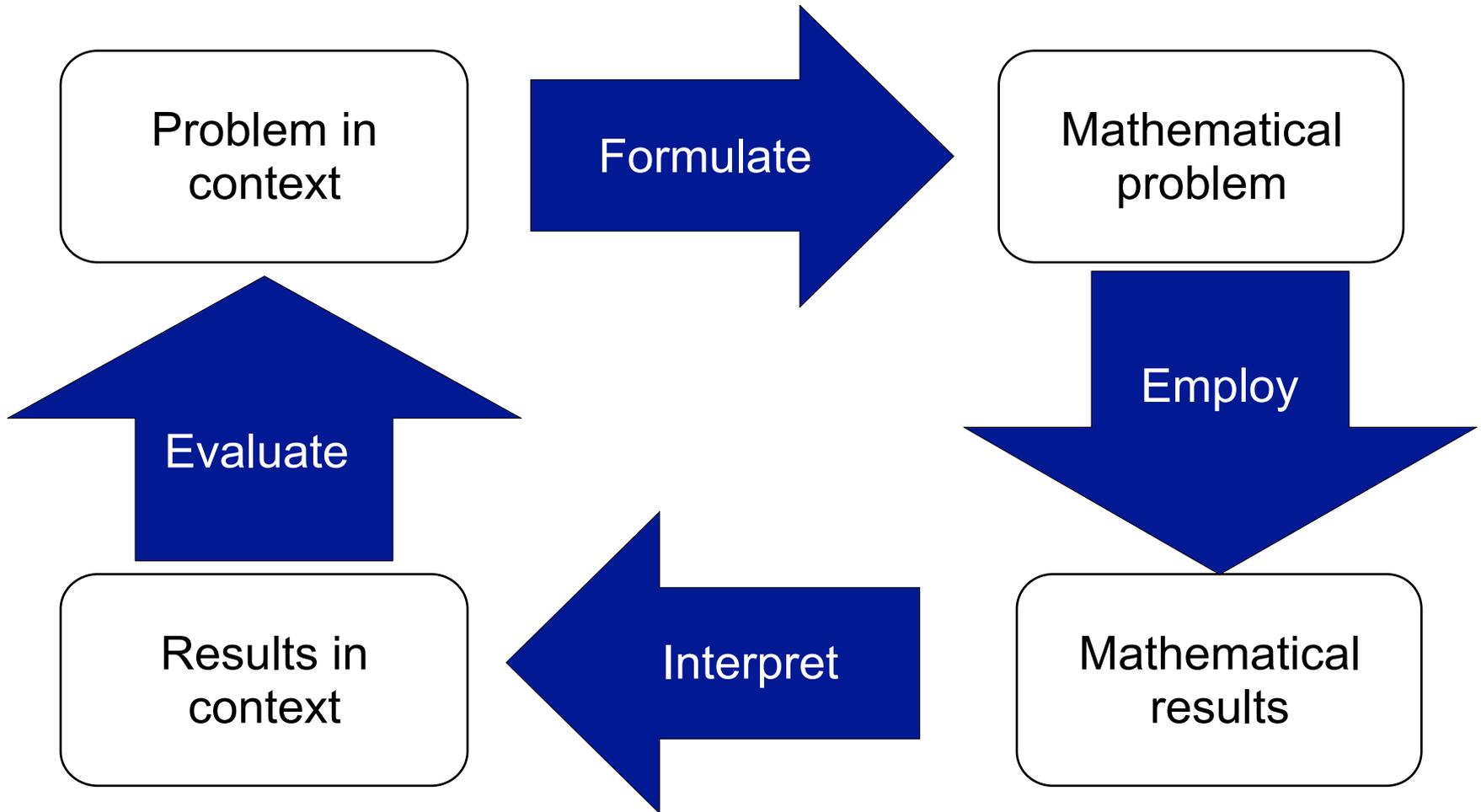
- **Modelling – what does it look like (11-16)?**
 - A 1-lesson modelling task
 - A 4-lesson modelling task
 - A 3-week modelling project
 - Modelling subskills
- **Supporting Modelling**
 - Professional Development and Pedagogy
 - Assessment
- **Why doesn't Modelling happen in our classrooms?**
 - Strategies for System Change



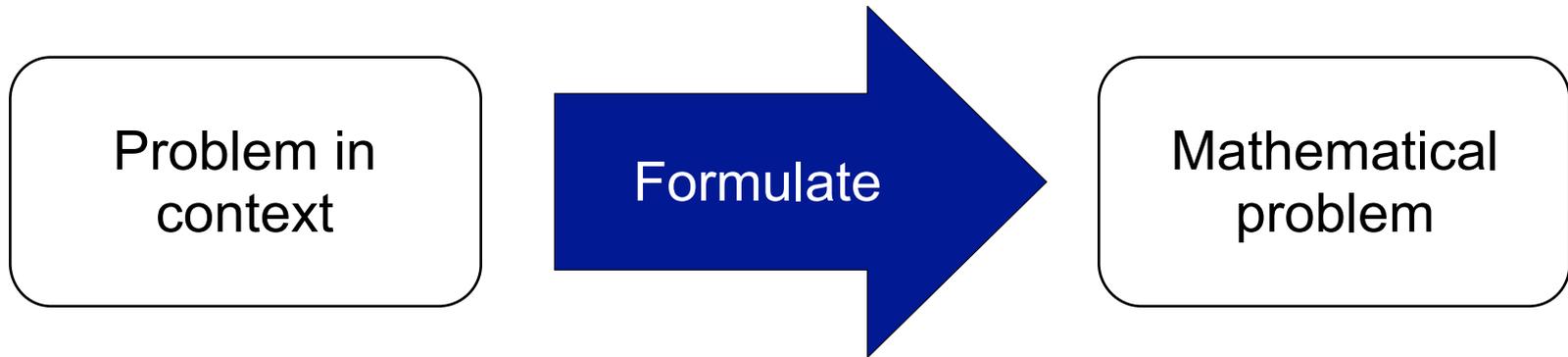
**Modelling: what does it look like
In secondary classrooms?**

Mathematical literacy (PISA, 2015)

“The modelling cycle is a central aspect of the PISA conception of students as active problem solvers”



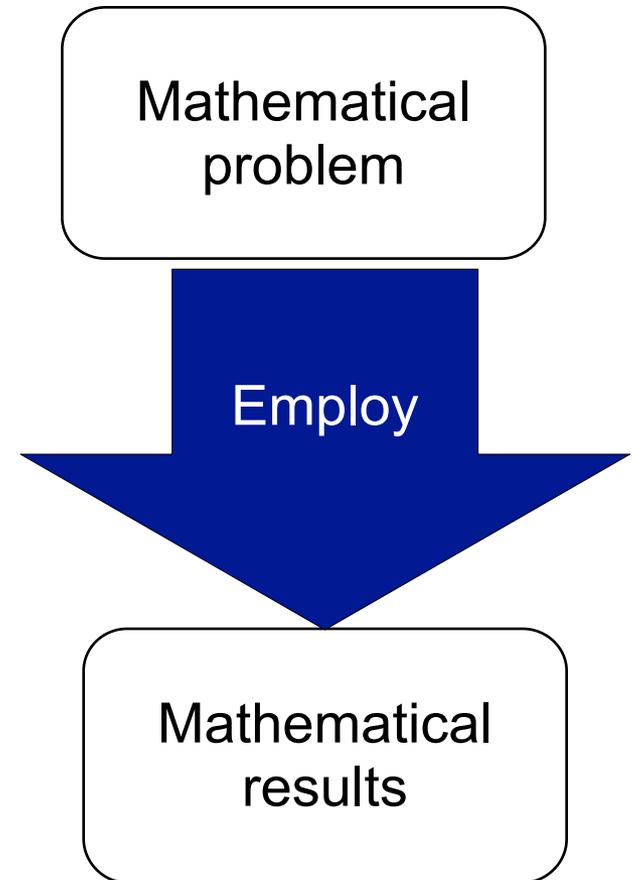
Formulating situations mathematically



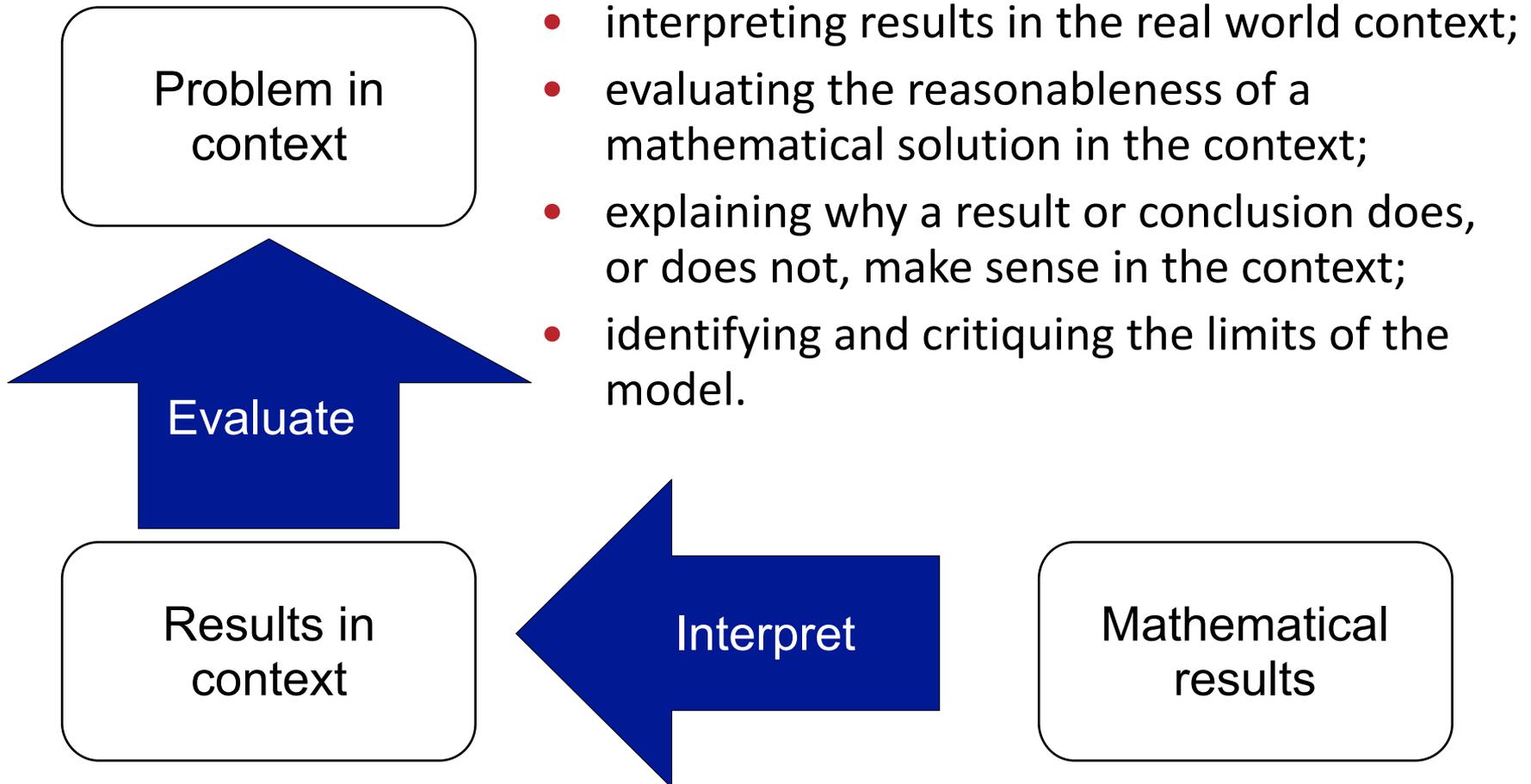
- Identify accessible questions that may be tackled
- Make suitable assumptions to simplify a situation
- Represent a situation mathematically
- Identify significant variables
- Generate relationships between variables
- Making connections with known problems or mathematical concepts, facts, or procedures

Employing concepts, facts, procedures and reasoning

- Select appropriate mathematical concepts and procedures
- Plan an approach
- Carry out the plan, monitoring progress and changing direction where necessary.



Interpreting and evaluating



A single lesson modelling task from Bowland Maths

Quantity

BOWLAND MATHS
www.bowlandmaths.org.uk

Imaginative resources for rich problem solving in secondary school Maths

Home About Bowland Maths Classroom projects Assessment tasks Professional development Lesson Study project

Welcome to Bowland Maths

Bowland Maths aims to make maths engaging and relevant to pupils aged 11-14, with a focus on developing thinking, reasoning and problem-solving skills. In these materials, the maths emerges naturally as pupils tackle problems set in a rich mixture of real-life and fantasy situations.

Classroom projects – the Case Studies

At the core of Bowland Maths are 26 extended mathematical investigations called Case Studies. Each case study includes teaching materials to support 3-5 lessons. Many, but not all, include ICT activities.

Assessment tasks

Bowland Maths includes a collection of over thirty 20-60 minute tasks with progression guides to support formative assessment of progress in mathematical reasoning and problem-solving skills.

Professional development

The Bowland materials demand different approaches to teaching, such as collaborative learning through discussion and reflection, self- and peer-assessment and the use of less structured tasks. These 7 video-based professional development modules help teachers explore these techniques.

Bowland Maths materials are free for non-commercial educational use, and can be viewed online or downloaded from this website.

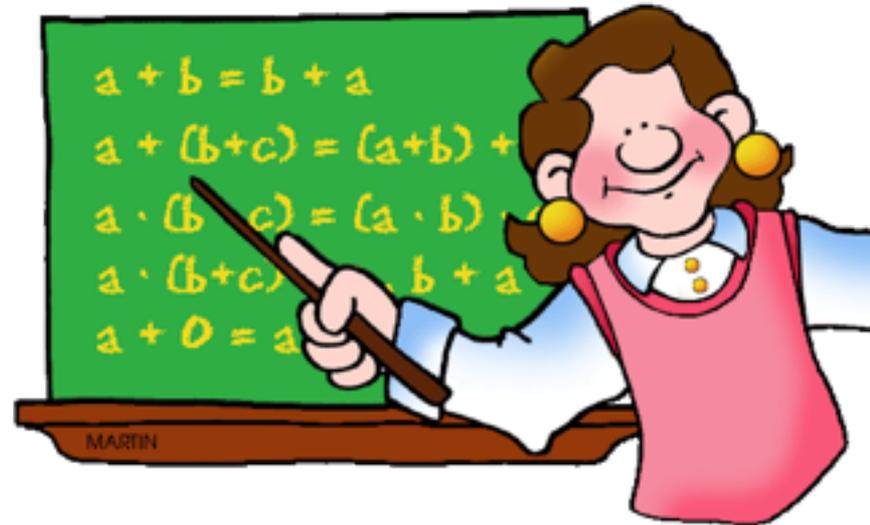
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<http://www.bowlandmaths.org.uk>

How many teachers?

There are about 60 million people in the UK.

About how many school teachers do we need?





-600

10/10

What were the students doing?

Identifying significant variables and making assumptions:

| | | |
|------------------------------|------------|-----|
| Size of population | 60,000,000 | p |
| How long do you go to school | 12 years | t |
| Average lifespan | 80 years | n |
| Size of class | 25 | c |

Deriving relationships and facts:

| | | |
|----------------------------------|-----------|-----------------------|
| Fraction of population at school | $1/8$ | $t \div n$ |
| School population | 7,500,000 | $p (t \div n)$ |
| Number of teachers | 300,000 | $p (t \div n) \div c$ |

In spite of errors, formulating processes are evident.

A 4-lesson modelling task from Bowland Maths

Data Handling

BOWLAND MATHS
www.bowlandmaths.org.uk

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<http://www.bowlandmaths.org.uk>

<http://www.bbc.co.uk/news/uk-15975720>

Every death on every road in Great Britain 1999-2010



Five a day



Did you know?



Explore your area



Time-lapse maps



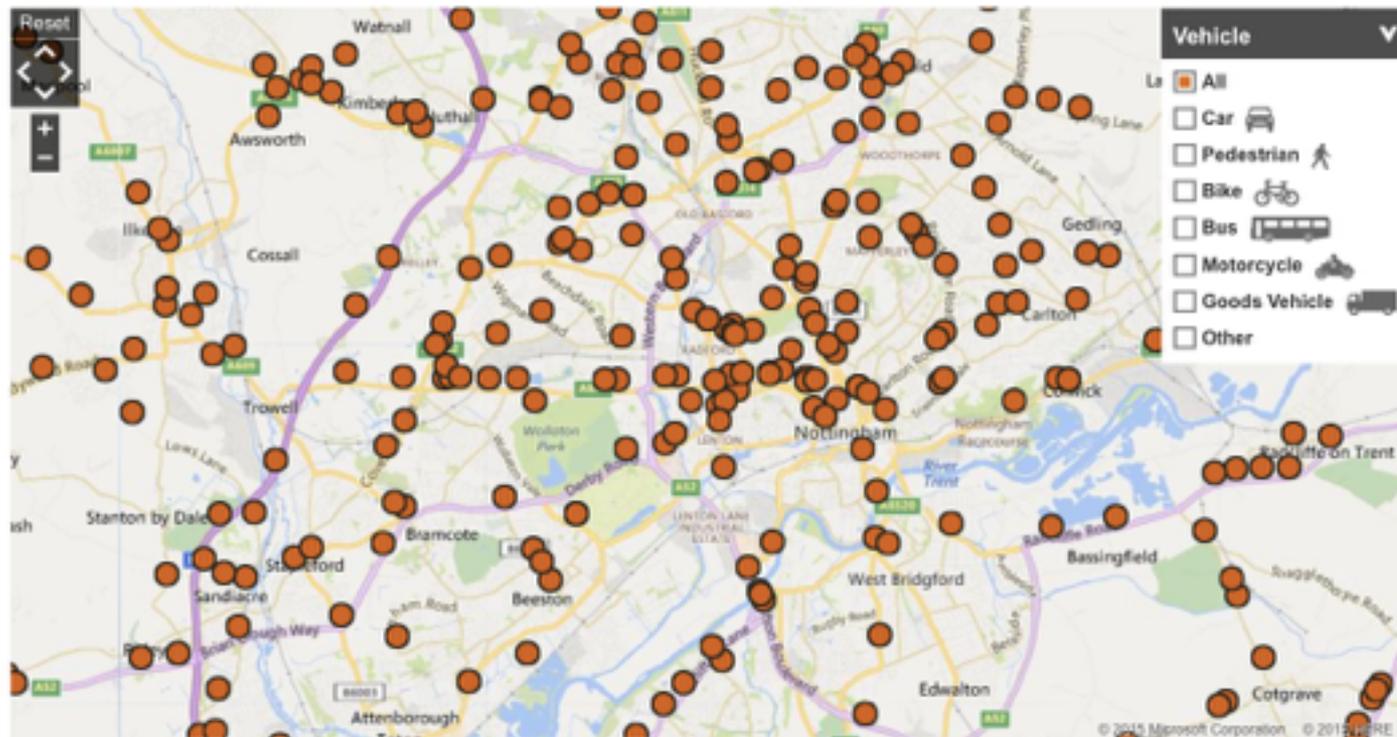
#crash24



Crash-proof cars

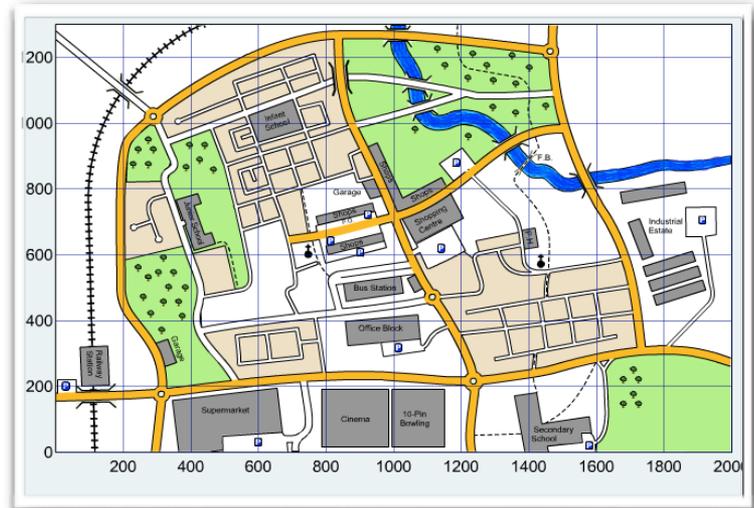
Search by Local Authority

From 1999-2010 131 people died on the roads in City of Nottingham local authority



Reducing Road accidents

- Students examine police reports, photographs and a map.
- They identify possible causes of road accidents and brainstorm remedies.



A simplified police report and matching photo:

| Police Record 1 | | | | | | | |
|--|------------|---------|--------------------------|-----------|---------------|----|----|
| Time | Day | | Date | | Year | | |
| 08.45 | Monday | | 8th September | | 1 | | |
| Location of accident | | | Map Reference | | | | |
| Ratrun Lane | | | East 475 m North: 1010 m | | | | |
| Name of casualty | | | Age of casualty | | | | |
| Sarah Crowe | | | 30 | | | | |
| Vehicle | Pedestrian | Cyclist | Car | Motorbike | Other vehicle | | |
| Road Conditions | Dry | | Wet | Snow | Frost/ice | | |
| Speed limit | 40 | 20 | 30 | 40 | 50 | 60 | 70 |
| Severity | Fatal | | Serious | | Slight | | |
| Description of accident | | | | | | | |
| Sarah was cycling along Ratrun Lane when the door of a parked car was opened suddenly. This knocked Sarah off her bike. She had a head injury. | | | | | | | |



Employing statistical representations

- Students are given a computer database of casualty reports.
- Students explore the data and try to find patterns in it:
- in accident locations, times, weather conditions, vehicle usage and so on.



Casualty report

Incident ID: 103

Age: 18 Female

Severity: Serious

Time: 19:20 Day: Sat

Date: 23 Dec Year: 4

Position - East (m): 700

Position - North (m): 225

Weather: Wet

Speed limit (mph): 50

Vehicle: Cycle

1 of 120



Map



Pie chart



Bar chart



Table



Choose data



Preparing a case

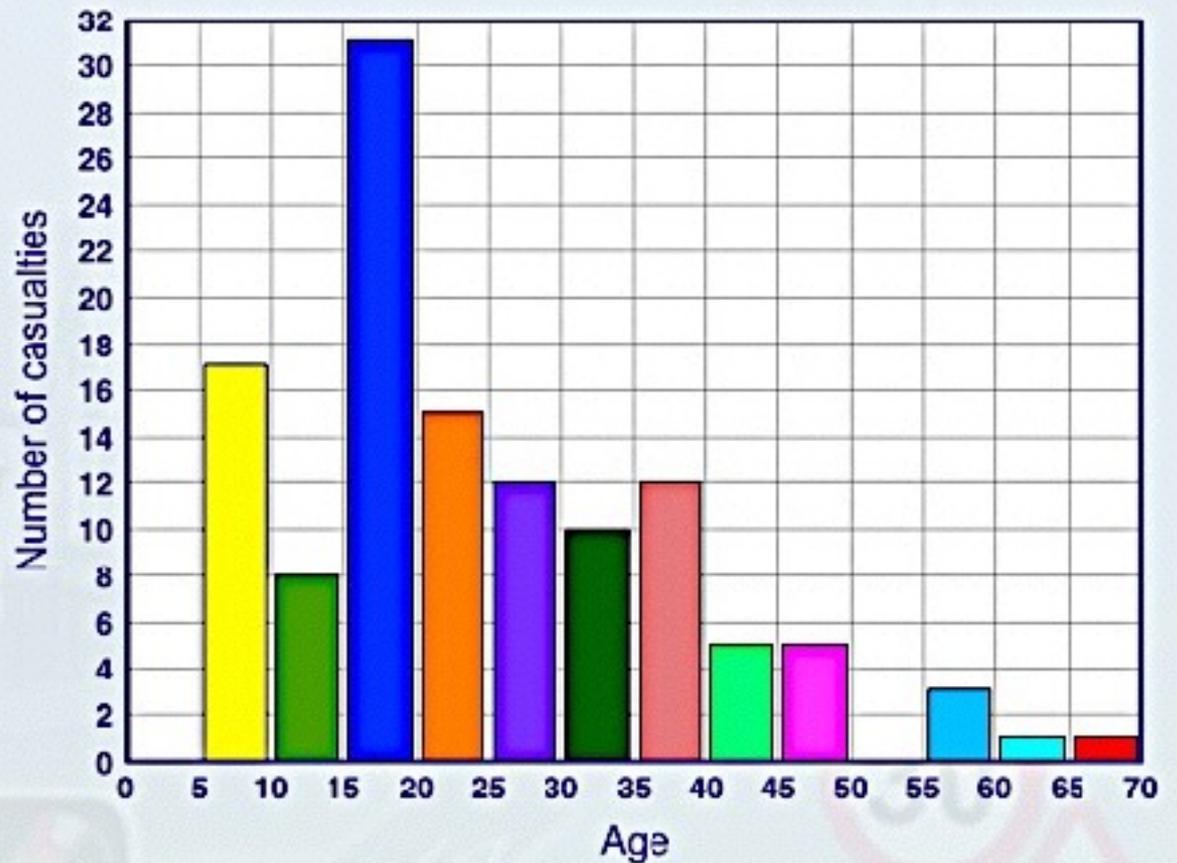
Each group of students is allocated a budget of £100,000 to spend on road improvements.

| | | |
|--|--|------------------------------------|
| <p>Road Safety campaign</p>  | <p>A poster and leaflet campaign can be effective when it targets a particular cause of accidents. You will need to describe</p> <ul style="list-style-type: none"> the focus of the campaign, the time of year it will appear, the type of person it will target. <p>You need to renew the campaign each year for it to continue having an effect.</p> | <p>£20,000 per year</p> |
| <p>Traffic lights</p>  | <p>Traffic lights can control the flow of traffic at junctions or other hazards, stopping some traffic while other traffic is allowed to go.</p> | <p>£30,000 per junction</p> |
| <p>Mini roundabout</p>  | <p>Mini-roundabouts are often only marked out with white paint. They are used on roads that have an average speed of 30mph or less. They are often used to reduce speed before a series of road humps.</p> | <p>£10,000</p> |
| <p>Large roundabout</p>  | <p>Large roundabouts are used to control the flow of traffic at junctions between major roads.</p> | <p>£40,500</p> |
| <p>Road narrowings</p>  | <p>Road narrowings slow traffic down by forcing one stream to give-way to the other. When they are on both sides of the road they are called chicanes or pinch points.</p> | <p>£10,000</p> |

| | | |
|--|--|-------------------------------|
| <p>Pelican crossing</p>  | <p>Pelican crossings control vehicle and pedestrian movements with traffic lights. Pedestrians must wait for the 'green man' before crossing the road</p> | <p>£18,000</p> |
| <p>Cycle lane</p>  | <p>Cycle lanes help keep bikes separate from other road users. They can be either on the side of the road or off-road.</p> | <p>£60 per metre</p> |
| <p>Traffic island and pedestrian refuge</p>  | <p>Traffic islands in the centre of a road to help reduce vehicle speeds and stop over-taking. If it includes a gap in the middle of the island it is called a refuge; it allows pedestrians to cross half the road at a time.</p> | <p>£3,000</p> |
| <p>Speed camera</p>  | <p>Speed cameras automatically photograph the number plates of drivers exceeding the speed limit. Many speeding drivers have been convicted by the photographic evidence.</p> | <p>£25,000</p> |
| <p>Speed humps</p>  | <p>Road humps can only be put on roads with speed limits of 30 mph or less. A series of humps should be about 50 metres apart and have a speed reducing feature at both ends, such as a road narrowing or mini roundabout.</p> | <p>£1,000 per hump</p> |
| <p>School crossing patrol</p>  | <p>A lollipop lady can help to ensure the safety of younger children. It is helpful if approaching traffic is slowed down by other measures.</p> | <p>£5,000 per year</p> |

Number of casualties vs. Age

| Age | Casualties |
|-------|------------|
| 0-4 | 0 |
| 5-9 | 17 |
| 10-14 | 8 |
| 15-19 | 31 |
| 20-24 | 15 |
| 25-29 | 12 |
| 30-34 | 10 |
| 35-39 | 12 |
| 40-44 | 5 |
| 45-49 | 5 |
| 50-54 | 0 |
| 55-59 | 3 |
| 60-64 | 1 |
| 65-69 | 1 |



Choose Variable



Map



Bar charts



Data table



Pie charts



Print (Ctrl-P)



Choose data

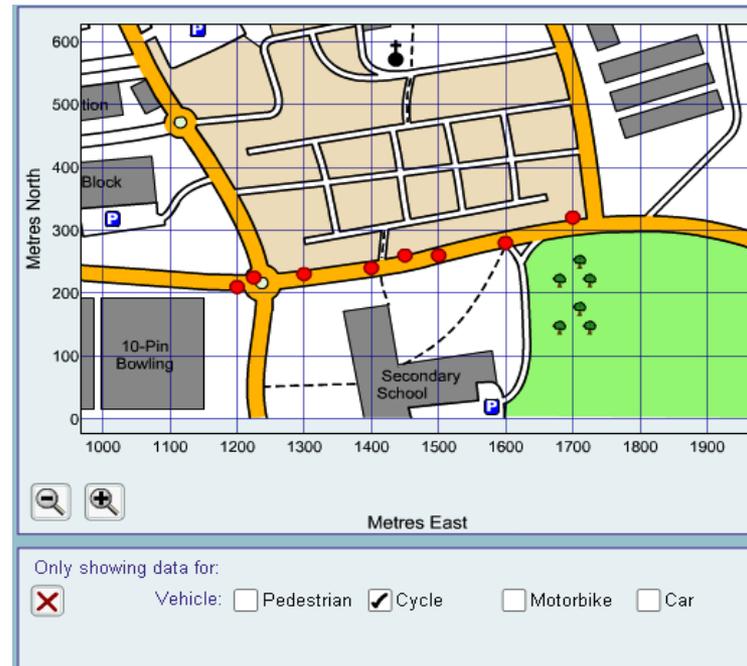
Sample student report

The problems we have found are that outside the school there are a number of accidents to cyclist leaving school.

To sort out this problem we are going to put in 500 metres of cycle lanes and tracks costing £30,000 out of our £100,000.

We could also add 8 road humps costing £8,000 To help the road humps we would need 2 traffic island and pedestrians refuge costing £6,000.

We should get 2 sets of pelican crossing costing £36,000 coming to a total cost of £80,000 this should reduce the accident rate by half. Hopefully helping save pupils lives on bikes every year.

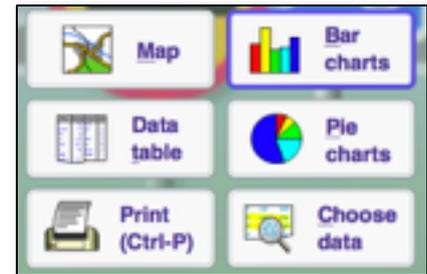


(Student work)

Choose variable to plot...

| | |
|-----------------|-----------------|
| Year [1] | Day of week [2] |
| Month [3] | Hour [4] |
| Age [5] | Sex [6] |
| Vehicle [7] | Weather [8] |
| Speed limit [9] | Severity [10] |

Modelling with Data



Formulating

- **Identify specific questions:** “What is relationship between number of accidents and ...?”
- **Make suitable assumptions:** “Let’s assume that the accidents are due to ...”
- **Represent mathematically:** “Which representation best shows ... ?”
- **Identify significant variables:** “Number of fatal accidents, time of day, age, weather..”
- **Generate relationships:** “Older people seem to have fewer accidents.”

Employing

- **Select and use appropriate mathematics:**
“How long is this road? What will a cycle lane cost”
- **Make and carry out a plan, monitor progress**
“Let’s find three big causes and tackle them.”

Interpreting and Evaluating

- **Interpreting results:** “About how many lives will this plan save?”
- **Evaluating the solution:** “Is John’s plan convincing?”

A 3-week
project
from

Numeracy
through Problem
Solving

Geometry



www.mathshell.com/materials

Numeracy through problem solving (1987-9)

- **Rationale:**

- The teaching, learning and assessment of numeracy through solving real problems
- Students as designers, planners, decision-makers.
- Students put their designs and plans *into action*.

- **Teaching time:**

- Each module:
10-20 hours over 3-6 weeks.

- **Certification at 3 levels:**

- Basic; Standard; Extension
- Teacher assessed
+ written exam papers
at higher levels.



Numeracy through problem solving (1987-9)

- **Design a Board Game**
 - Geometry, probability
- **Plan a Trip**
 - Money, time, distance
- **Be a Shrewd Chooser**
 - Money, data handling,
- **Be a Paper Engineer**
 - 3D geometry, algebra
- **Produce a Quiz Show**
 - Scheduling, fairness



Numeracy through Problem Solving

| | Number | Algebra | Shape and Space | Handling data |
|---------------------|--------|---------|-----------------|---------------|
| Design a Board Game | | | *** | * |
| Be a Paper Engineer | | * | *** | |
| Plan a Trip | *** | | * | * |
| Produce a Quiz show | *** | | * | * |
| Be a Shrewd Chooser | * | * | | *** |

Mathematics through Problem Solving

| | Number | Algebra | Shape and Space | Handling data |
|----------------------|--------|---------|-----------------|---------------|
| Design a Board Game | | | *** | * |
| Be a Paper Engineer | | * | *** | |
| Plan a Trip | *** | | * | * |
| Produce a Quiz show | *** | | * | * |
| Be a Shrewd Chooser | * | * | | *** |
| Patterns and Numbers | * | *** | | |
| Functions and Graphs | | *** | | |

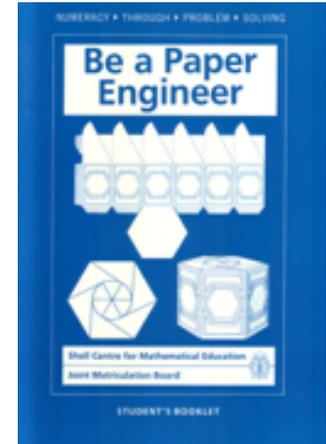
Each module has 4 stages

- **Stage 1: Looking at existing examples**
 - Students analyse existing products or processes and critique them
 - Students identify criteria/ structures for successful products
- **Stage 2: Planning an approach**
 - Students brainstorm ideas, select one to work on, study the techniques they need and plan their work.
- **Stage 3: Carrying out the approach**
 - Students carry out their plan, make a prototype.
- **Stage 4: Presenting and evaluating the outcomes**
 - Students make/enact the product/process.
 - Students test and evaluate each others' work.



Stage 1: Looking at examples

- Students, share the work of making 30 boxes, envelopes, pop-up cards.
- They classify them according to their structure.
- They bring in outside examples from home as well.



8. A Barbecue Invitation



Cut round the outline of the card.
Score along the fold lines.
Fold along the fold lines, and then try to make the hill and valley folds go the right ways:



12. The Ghost

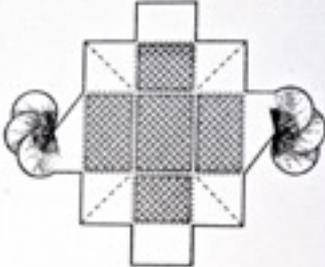


Cut out the base and fold it in half.
Cut out the "ghost" piece. Remember to cut along the top of the wall between the fold lines. (From X to Y in the above diagram.)
Score then make the fold lines. Be careful with the hill and valley folds.
Match tabs A and B to the base and glue in place.

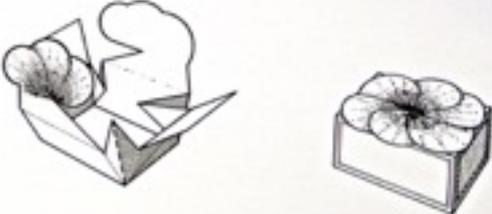


Now open the card and watch the ghost rise!

24. The Flower Box

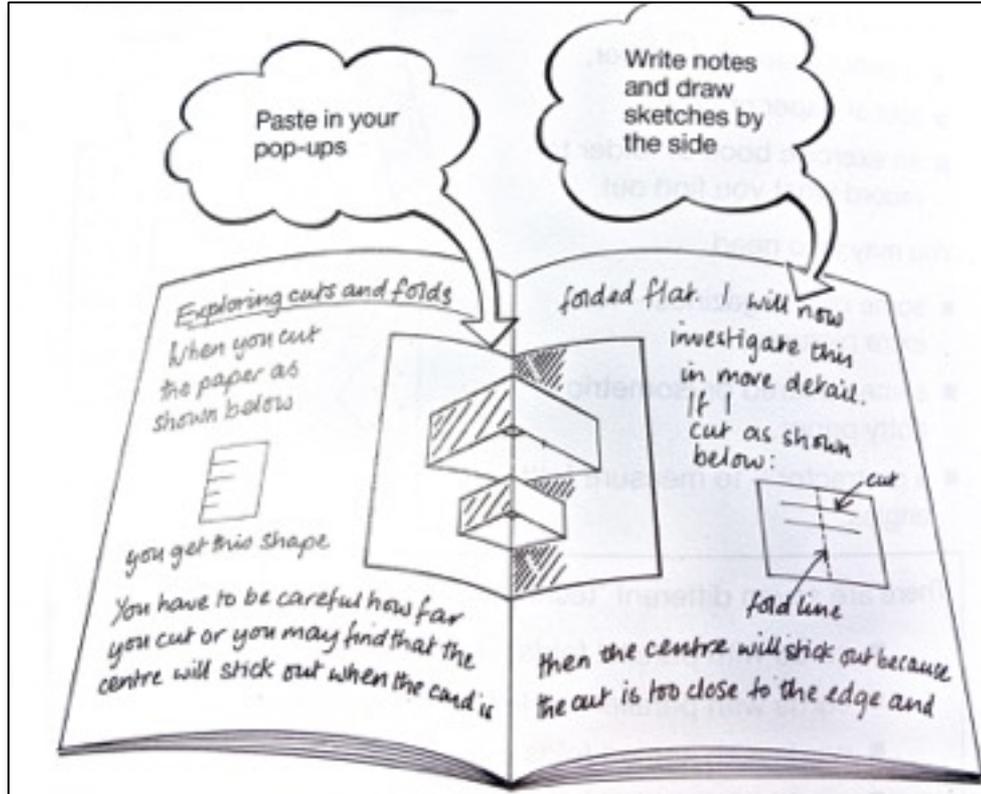


No glue is needed for this box. Score along all the fold lines. Make the folds, but be careful with the hill and valley folds.
The two halves of the flower slot together.

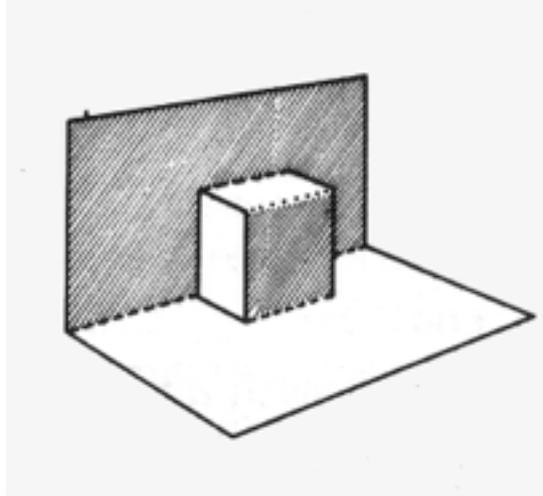


Stage 2: Exploring techniques

- Students select a type of box or card they are interested in and explore the mathematical principles involved in more depth.
- They tackle a series of investigations and challenges and write up their findings.

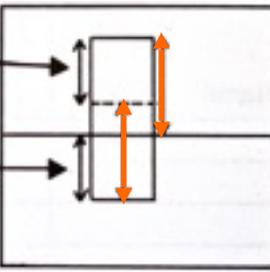


Stage 2: Exploring techniques



The cards which will shut properly are: $A+B$

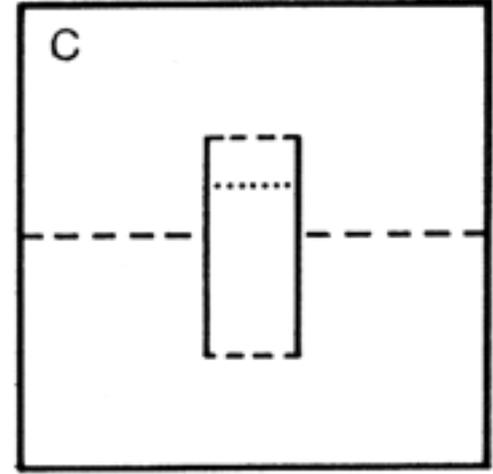
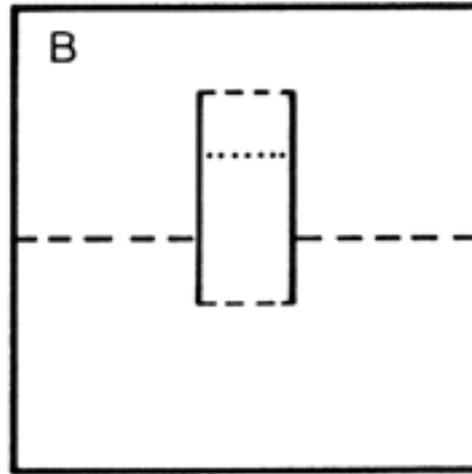
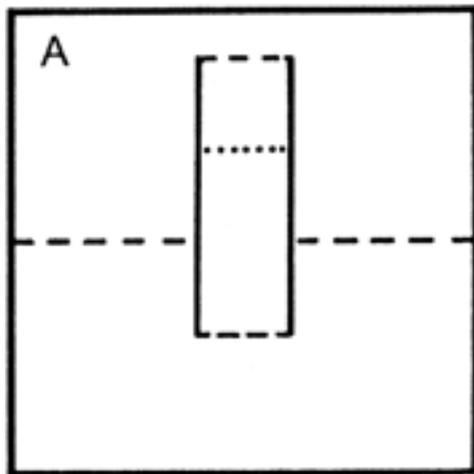
The reason why I can tell is that the length of this



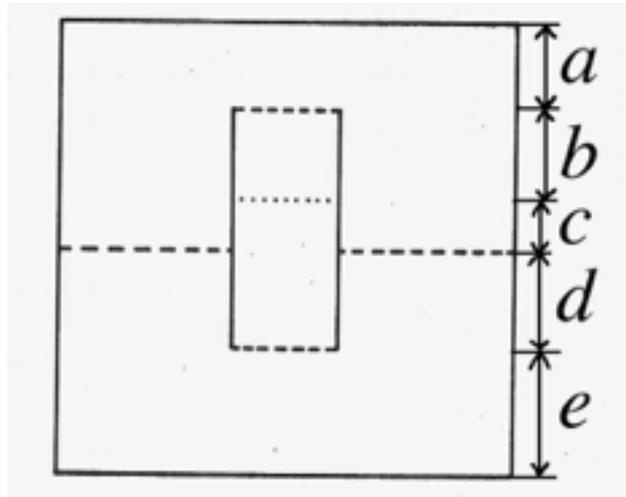
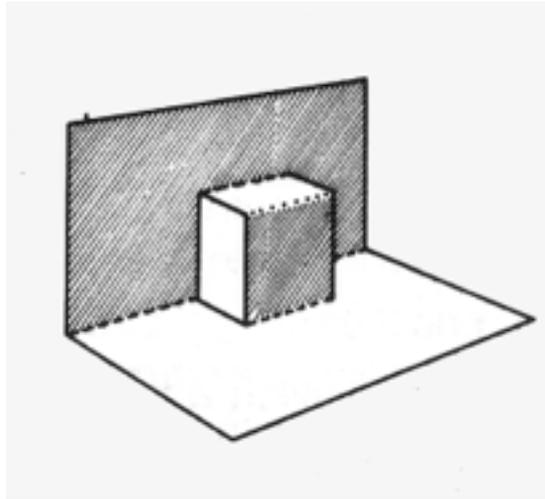
must be the same length as this

or the orange arrow must be the same length as the other arrow

Which of these cards will work properly? How can you tell?



Stage 2: Exploring techniques



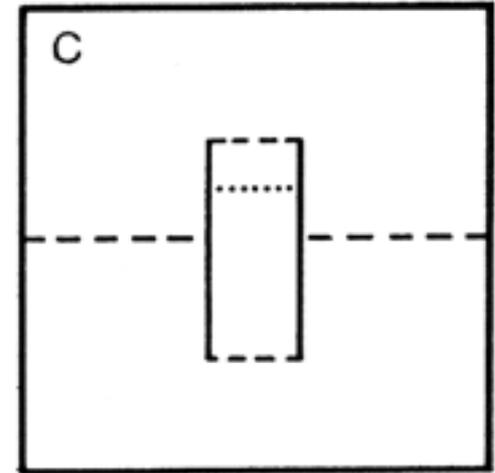
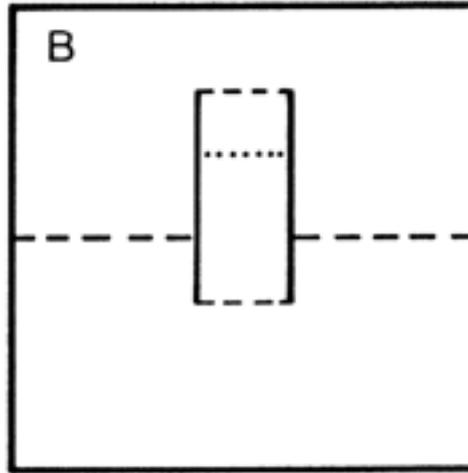
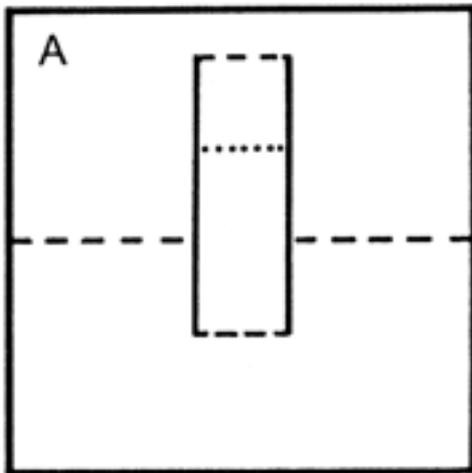
$$a+b+c=d+e$$

$$b=d$$

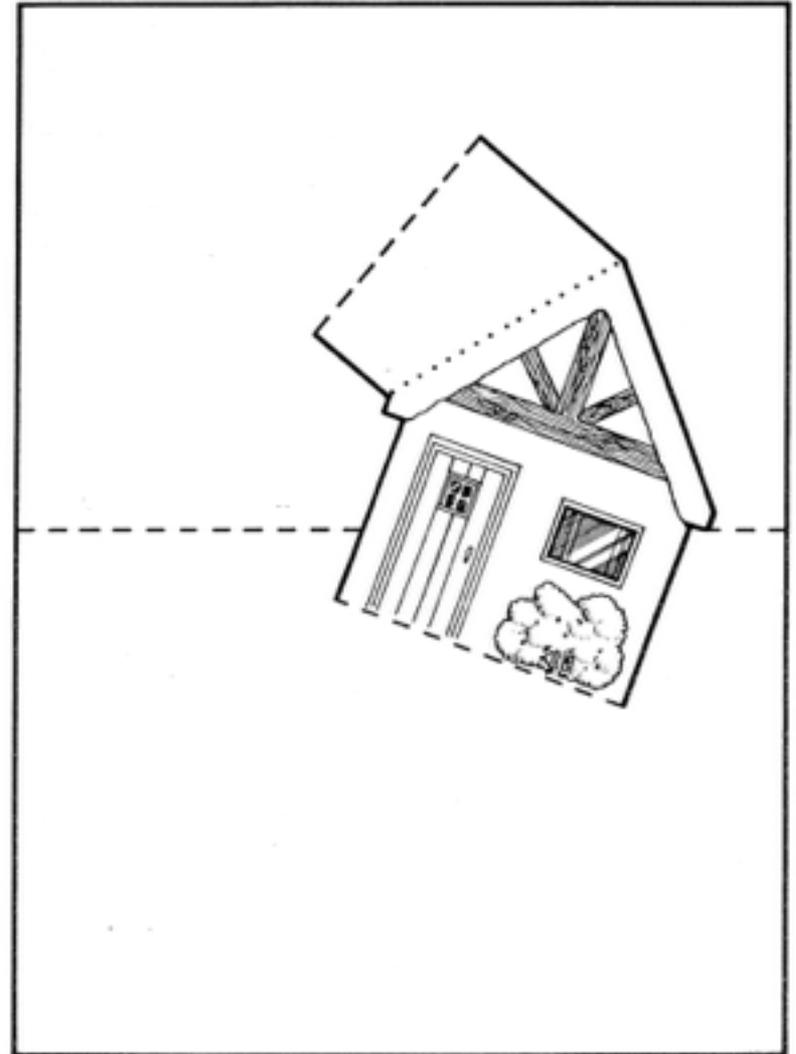
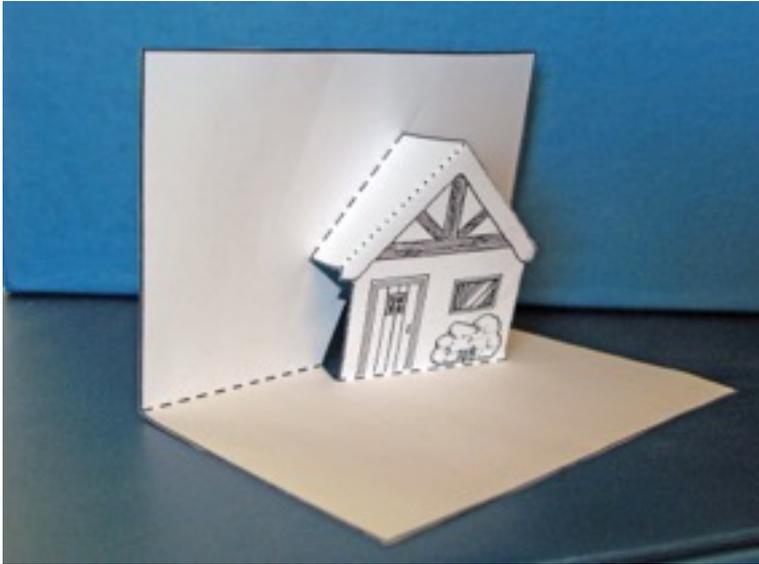
$$b < a$$

$$c+d < e$$

Which of these cards will work properly? How can you tell?

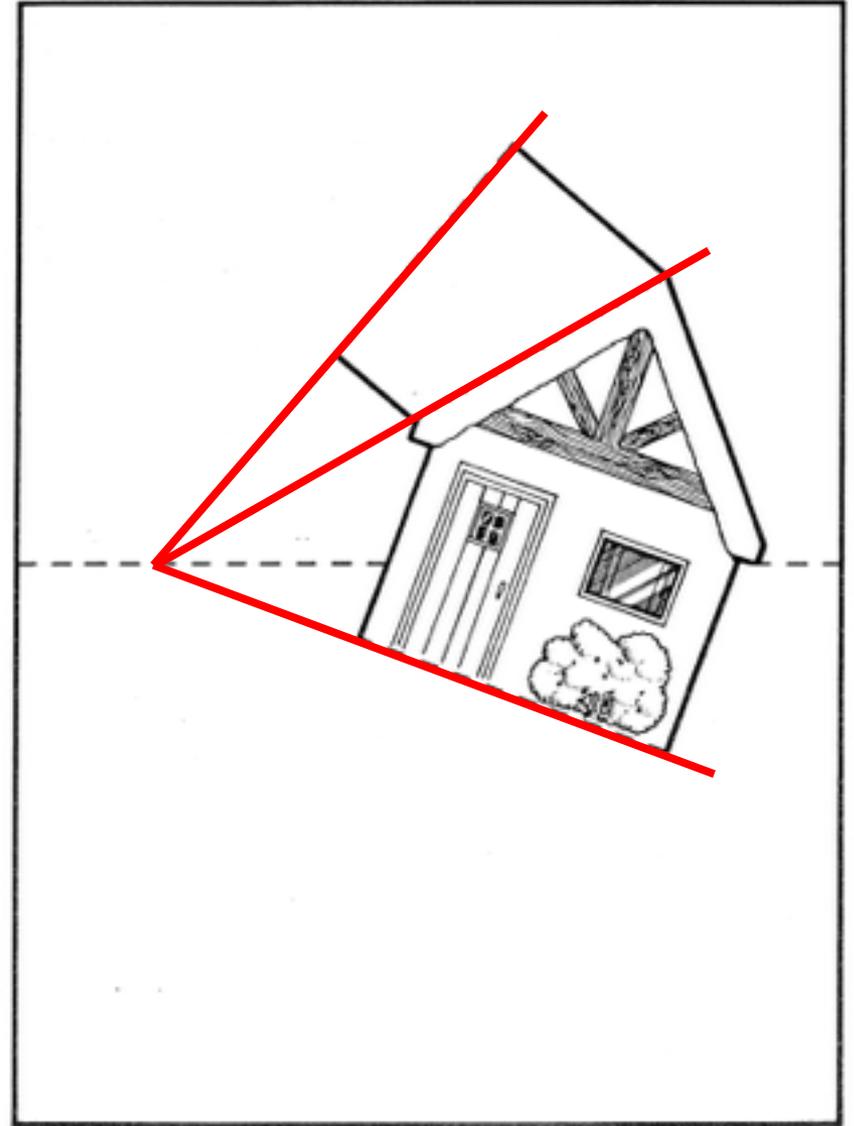
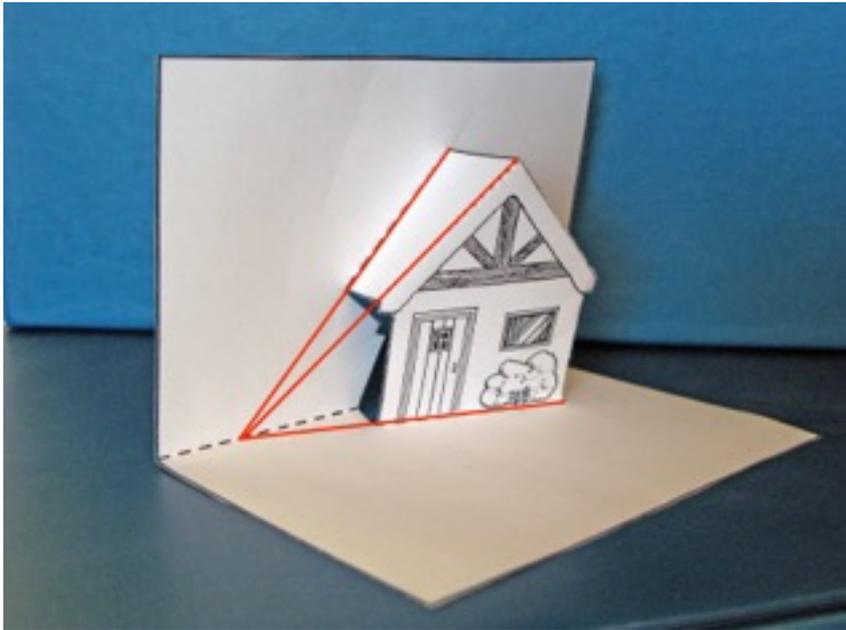


Stage 2: Exploring techniques



- If this card is to work properly, what can you say about:
- the directions of the fold lines;
- the angles between the fold lines?

Stage 2: Exploring techniques



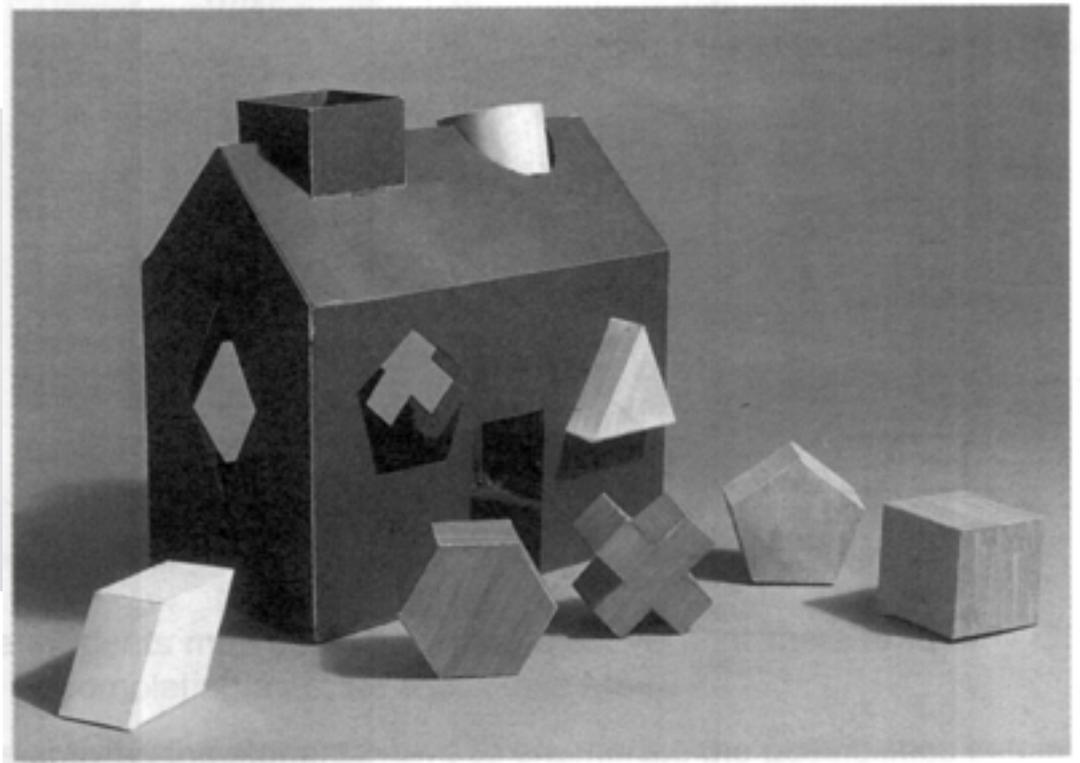
Stage 3: Making your own original

- Students brainstorm ideas for their own product.
- They then each make a rough version, followed by a final version



Stage 4: Going into production

- Students devise step-by-step instructions for making their product.
- Students produce and test their instructions, by asking someone else to follow them.



Modelling with Geometry

Formulating

- **Identify specific questions:** “How can I make a card that does this..?”
- **Make simplified drawings:** “Let’s study its structure...”
- **Represent mathematically:** “How can we draw this 3D shape in 2D ... ?”
- **Identify significant variables:** “Which lengths / angles are important here? ..”
- **Generate relationships:** “How does this length depend on this one?”

Employing

- **Make & carry out a plan** “Can we draw *before* making cuts?”
- **Select appropriate maths:** “Can we use the principles we discovered?”

Interpreting and Evaluating

- **Interpreting results:** “Can you interpret John’s instructions for making it?”
- **Evaluating the solution:** “How can you improve John’s design?”

Issues that led to lack of use

- **Teacher expectations**

- Teachers found that it took them outside what they understood to be Mathematics.

- **Demand on teachers**

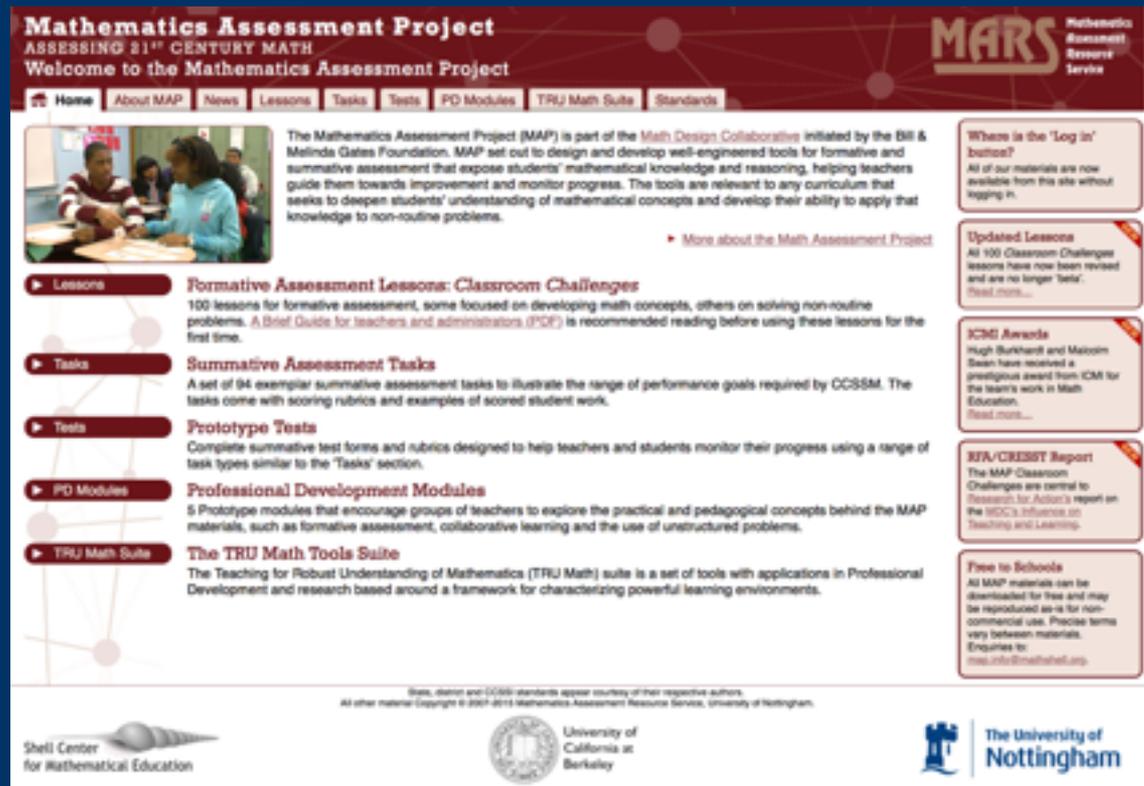
- 5 x 10-20 hours was too much for many teachers.
- Pedagogy was so different to what they had been used to.

- **Teachers tended to use it mainly with 'low attainers'**

- The roots of the scheme in 'numeracy', together with its emphasis on practical activity made some teachers reluctant to use it with more able students.

- **Teachers used it for cross curricular work**

Pedagogies for Modelling



Mathematics Assessment Project
ASSESSING 21ST CENTURY MATH
Welcome to the Mathematics Assessment Project

Home About MAP News Lessons Tasks Tests PD Modules TRU Math Suite Standards

 The Mathematics Assessment Project (MAP) is part of the [Math Design Collaborative](#) initiated by the Bill & Melinda Gates Foundation. MAP set out to design and develop well-engineered tools for formative and summative assessment that expose students' mathematical knowledge and reasoning, helping teachers guide them towards improvement and monitor progress. The tools are relevant to any curriculum that seeks to deepen students' understanding of mathematical concepts and develop their ability to apply that knowledge to non-routine problems.

[More about the Math Assessment Project](#)

- Lessons**
Formative Assessment Lessons: Classroom Challenges
100 lessons for formative assessment, some focused on developing math concepts, others on solving non-routine problems. A [Brief Guide for teachers and administrators \(PDF\)](#) is recommended reading before using these lessons for the first time.
- Tasks**
Summative Assessment Tasks
A set of 94 exemplar summative assessment tasks to illustrate the range of performance goals required by CCSSM. The tasks come with scoring rubrics and examples of scored student work.
- Tests**
Prototype Tests
Complete summative test forms and rubrics designed to help teachers and students monitor their progress using a range of task types similar to the 'Tasks' section.
- PD Modules**
Professional Development Modules
5 Prototype modules that encourage groups of teachers to explore the practical and pedagogical concepts behind the MAP materials, such as formative assessment, collaborative learning and the use of unstructured problems.
- TRU Math Suite**
The TRU Math Tools Suite
The Teaching for Robust Understanding of Mathematics (TRU Math) suite is a set of tools with applications in Professional Development and research based around a framework for characterizing powerful learning environments.

Where is the 'log in' button?
All of our materials are now available from this site without logging in.

Updated Lessons
All 100 Classroom Challenge lessons have now been revised and are no longer 'beta'.
[Read more...](#)

ICMI Awards
Hugh Burkhardt and Malcolm Swan have received a prestigious award from ICMI for the team's work in Math Education.
[Read more...](#)

RIA/CREST Report
The MAP Classroom Challenges are central to [Research for Action's](#) report on the RSC's influence on Teaching and Learning.

Free to Schools
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Shell Center for Mathematical Education | University of California at Berkeley | The University of Nottingham

<http://map.mathshell.org>

New role for the teacher

Transmission

- **Maths:**
body of knowledge to cover
- **Learning:**
individual listening and imitating
- **Teaching:**
linear explaining

Discovery

- **Maths:**
students create maths
- **Learning:**
individual exploration and reflection
- **Teaching:**
provide stimulating environment to explore, sequences activities and facilitates.

Collaborative

- **Maths:**
teacher and students co-create maths
- **Learning:**
collaborative learning through discussion
- **Teaching:**
challenges, non-linear dialogue exploring meanings and connections.

Mathematics Assessment Project

100 sample lessons + PD Support
Grades 6, 7, 8 and High School



MAP Professional Development Modules

▼ Supporting 21st Century Math Teaching

- ▶ 1: Formative Assessment
- ▶ 2: Concept Development Lessons
- ▶ 3: Problem Solving Lessons
- ▶ 4: Improving Learning Through Questioning
- ▶ 5: Students Working Collaboratively

Formative assessment = Adaptive teaching

Students and teachers

Using evidence of learning

To adapt teaching and learning

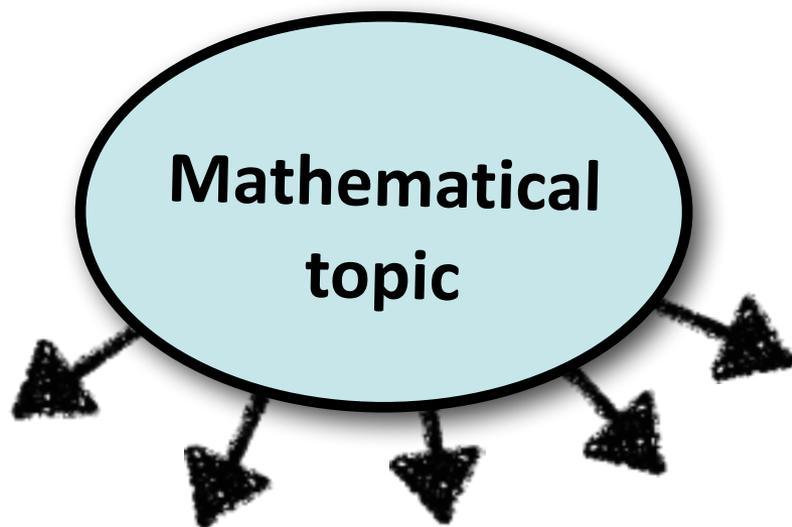
To meet immediate needs

Minute-to-minute and day-by-day

(Thompson and Wiliam, 2007)

Establishing the purpose of the lesson

Concept development



Modelling processes



Cats and Kittens



Cats can't add but they can multiply!

**In just 18 months, this female cat can have
2000 descendants**

Make sure your cat cannot have kittens

Is this figure of
2000
realistic ?

Number of
kittens in a
litter

Usually
4 to 6

Age at
which a
female cat
gets
pregnant

About 4
months.

Age at
which a
female cat
no longer
has kittens

About
10 years

Average
number of
litters a
female cat
can have in
one year

3

Length of
pregnancy

About 2
months

Before the lesson...

- **Problem presented to students**
- **Students tackle the problem unaided**
- **Teacher assesses student work and prepares qualitative feedback**

| Issue | Suggested questions and prompts |
|--|--|
| Has difficulty starting | <ul style="list-style-type: none"> • Can you describe what happens during first five months? |
| Does not develop suitable representation | <ul style="list-style-type: none"> • Can you make a diagram or table to show what is happening? |
| Work is unsystematic | <ul style="list-style-type: none"> • Could you start by just looking at the litters from the first cat? What would you do after that? |
| Develops a partial model | <ul style="list-style-type: none"> • Do you think the first litter of kittens will have time to grow and have litters of their own? What about their kittens? |
| Does not make clear or reasonable assumptions | <ul style="list-style-type: none"> • What assumptions have you made? Are all your kittens are born at the beginning of the year? |
| Makes a successful attempt | <ul style="list-style-type: none"> • How could you check this using a different method? |

Typical Lesson Structure

1. Introduction

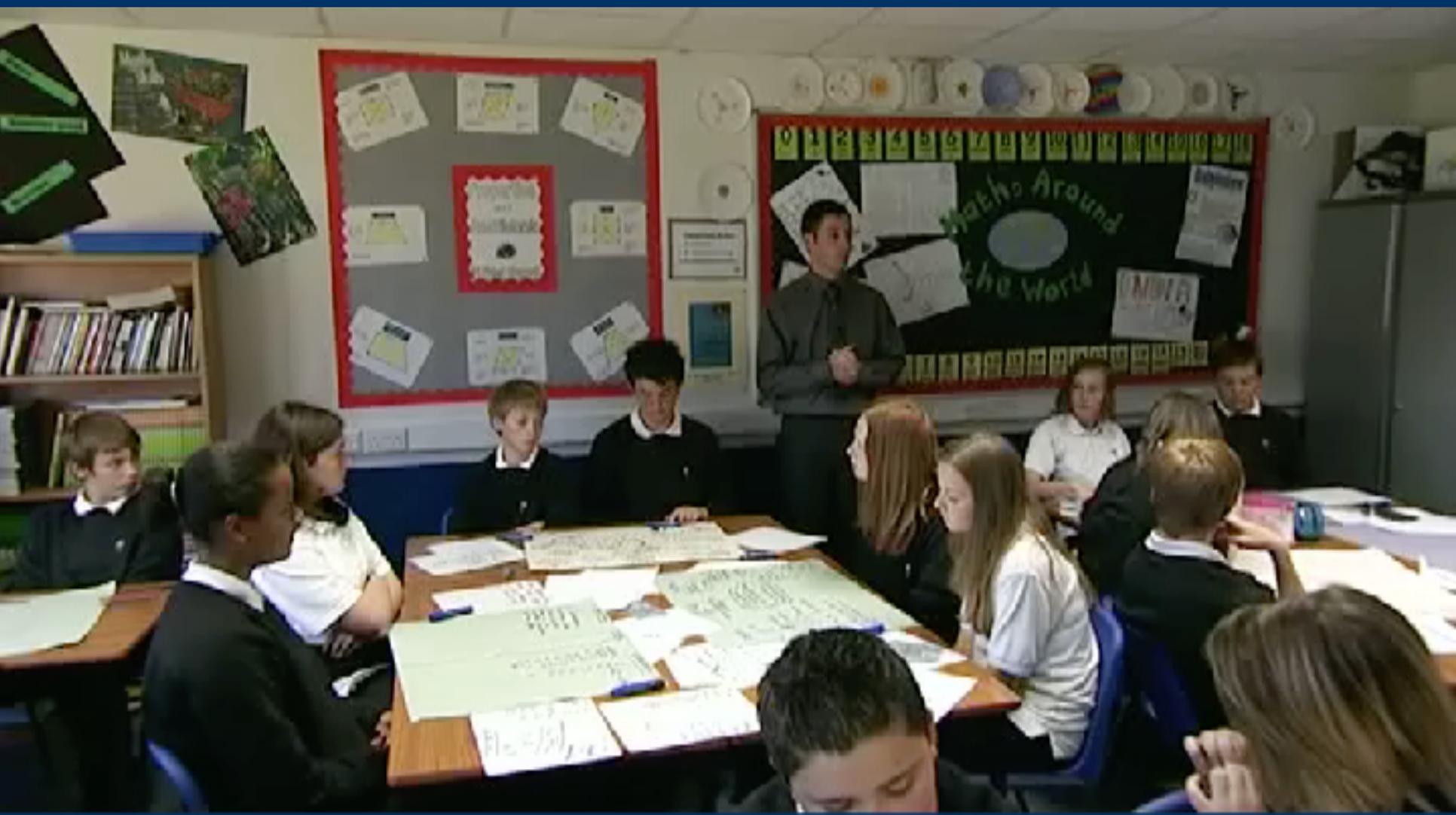
- Teacher re-introduces the main task for the lesson. Students respond to the prepared questions.

2. Group work: Comparing strategic approaches

- Students discuss the work of each individual, then produce a poster showing a joint solution. This promotes peer assessment.

3. Inter-group discussion

- Groups compare approaches and justify their own.



Students may not choose powerful approaches.

“Since problems can often be approached in different ways, and this sometimes means using different mathematics, care would have to be taken to ‘shape’ the approach of the students so that the desired mathematics is being used.”

(Additional Mathematics - AQA GCSE Teacher’s Guide, 2008, p10)

But:

- If we tell them to try a particular approach, opportunities for decision-making are taken away from the student. The problem solving lesson becomes an exercise in imitating a method.
- So how can we introduce more powerful approaches, without also removing student decision-making?

Typical Lesson Structure

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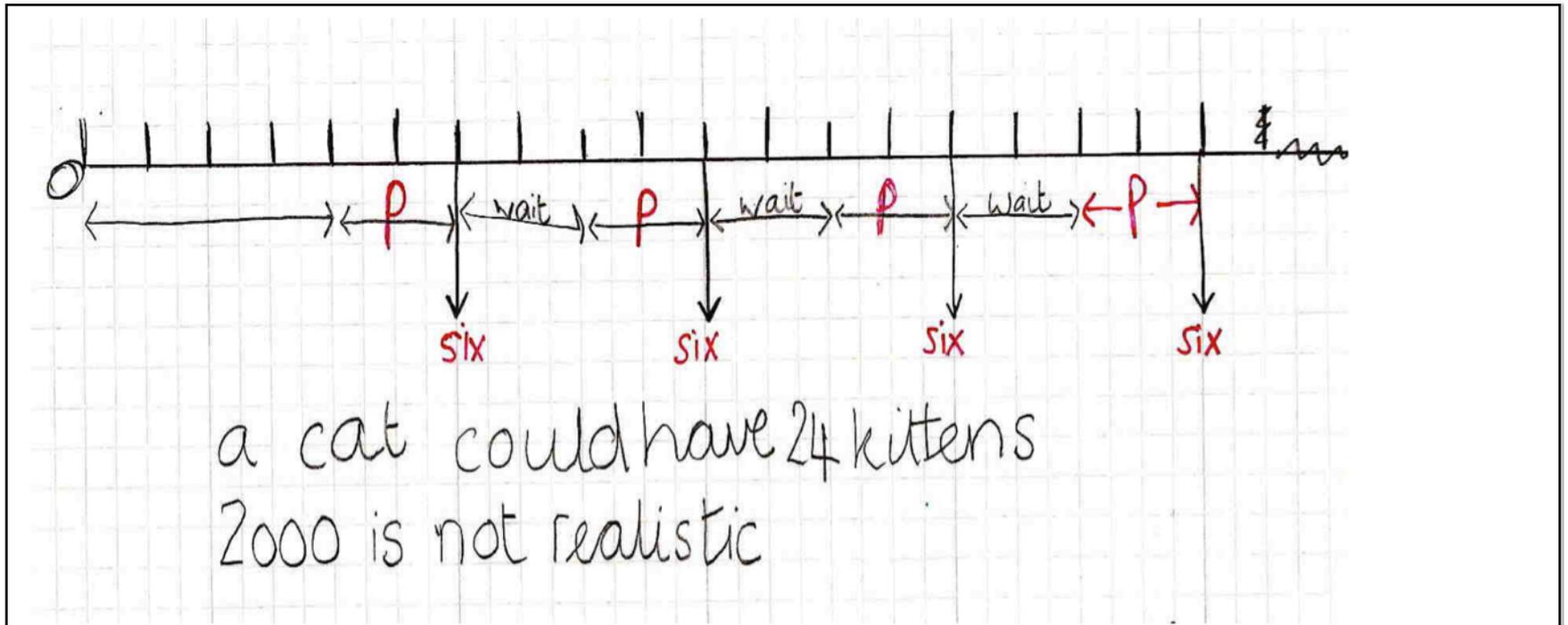
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- Groups compare approaches and justify their own.

4. Group work: Critiquing pre-prepared sample student work

- Students discuss approaches they may not have considered

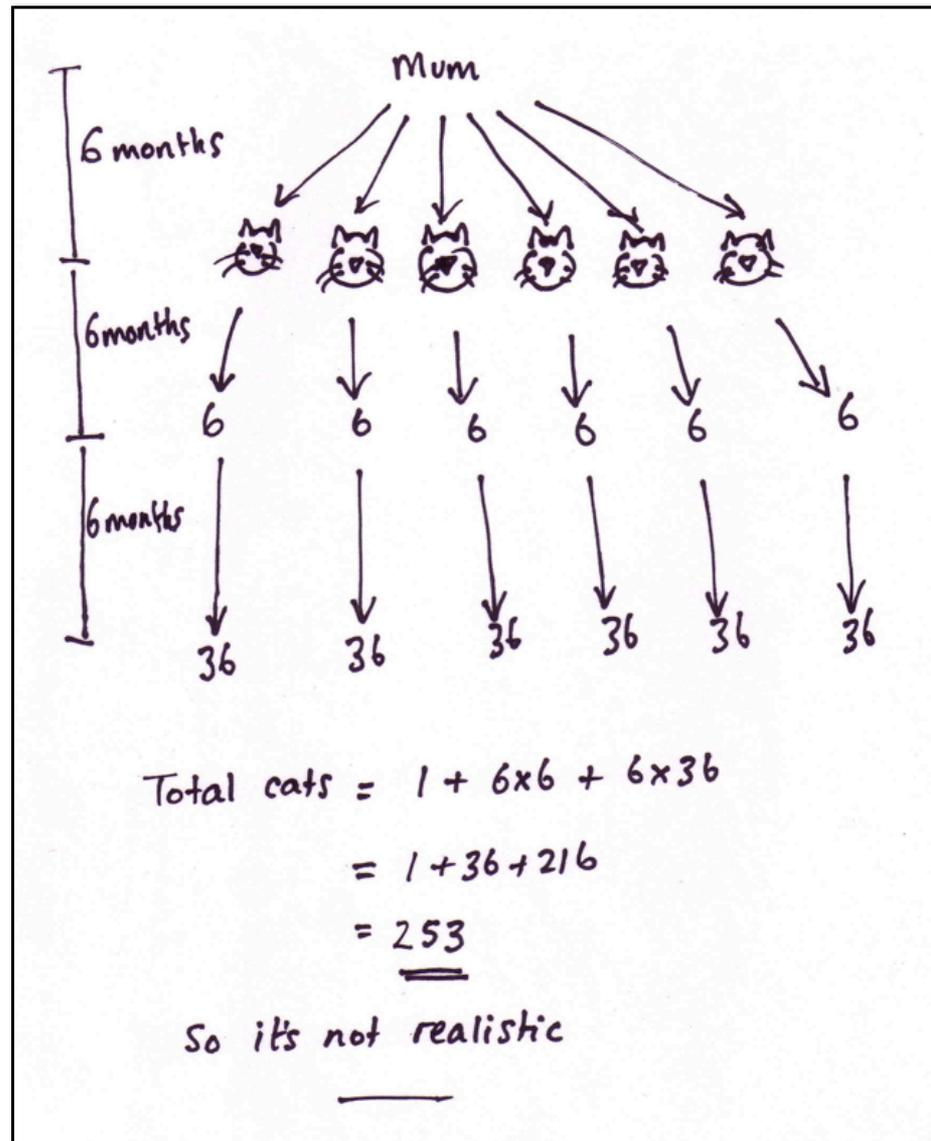
Sample student work: Alice



- What assumptions has Alice made?
- What has Alice forgotten?

Sample student work: Wayne

- What assumptions has Wayne made?
- What has Wayne forgotten?



Possible uses of “sample student work”

- To encourage a student that is stuck in one line of thinking to consider others
- To enable students to make connections
- To compare alternative representations
- To compare hidden assumptions and their effect
- To encourage metacognitive behavior
- To draw attention to common errors
- To encourage criticality without fear of criticism
- To become more aware of valued criteria for assessment

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5. Group work: Refining solutions

- Students improve their solutions to the problem

6. Whole class discussion: a review of learning

- Discussion of processes; implications of assumptions, alternative representations



Supporting Modelling



The Case Studies and Mathematics

Where is the maths in these Case Studies?



Tackling unstructured problems

Do I just stand back and watch or intervene and tell them what to do?



Fostering and managing collaborative work

How can I get pupils to stop talking and start discussing?



ICT: Using resources effectively

How do I get them to stop playing and start thinking?



Questioning and reasoning

How can I get my pupils to think, reason and explain?



Assessing the Key Processes

How do I assess problem-solving and mathematical reasoning?



Involving pupils in self and peer assessment

How can pupils help each other to progress in using Key Processes?

Alternative forms of PD

- **“Training” models**
 - Transmission of information by an ‘expert’.
 - Useful mainly for raising awareness of an initiative, but may feel alien to teachers.
- **“Experiential course” models**
 - Courses mediated by a provider, that offer teachers opportunities to explore ideas in their own classrooms and report back. May be accredited.
- **“Embedded” professional learning communities**
 - Teachers take over responsibility for setting their own research goals and collaboratively and systematically study them in their own classrooms. This may be informed by outside support from materials and/or invited ‘experts’.

Bowland Professional Development Modules

The projects and Mathematics

- Where is the maths?

Tackling unstructured problems

- Do I stand back or intervene and tell them what to do?

Fostering and managing collaborative work

- How can I get them to stop talking and start discussing?

ICT: Using resources effectively

- How can I get them to stop playing and start thinking?

Questioning and reasoning

- How can we ask questions that improve thinking and reasoning?

Assessing modelling processes

- How do I assess progress?

Involving students in self and peer assessment

- How can students help each other to progress?

Professional Development Modules

Introductory session:

- Teachers work on problems, discuss pedagogical challenges they present, watch video of other teachers using these problems and plan lessons.

Into the classroom:

- Teacher all teach the planned lessons.

Follow-up session:

- Teachers describe and reflect on what happened, discuss video extracts, and plan strategies for future lessons.

Summative Assessment tasks: Counting Trees



- The circles show **old** trees
- ▲ The triangles show **young** trees

Think of a method you could use to estimate the number of trees of each type.

Explain the method fully

Use your method to estimate the number of old trees and young trees

Progression Grids



Progression

| Formulate | Employ | Interpret and evaluate |
|--|---|---|
| <p>Incorrect assumptions. Assumes equal numbers of old/young trees.</p> <p>Uses inappropriate method. Attempts to count all trees.</p> | <p>Multiplies trees in one row by trees in one column.</p> <p>Ignores gaps / entire rows.</p> | <p>Does not consider reasonableness or check answer.</p> |
| <p>Assumes different numbers of old/young trees.</p> <p>Uses sampling, but sample is unrepresentative or too small.</p> | <p>Attempts to count trees systematically.</p> <p>E.g. Multiplies trees in one row by number of rows.</p> | <p>Makes an estimate.</p> <p>Answer is unreasonable due to method.</p> <p>Checks answer by repeating method.</p> |
| <p>Chooses appropriate sampling method.</p> | <p>Follows chosen method mostly accurately.</p> <p>Uses proportion to obtain an estimate.</p> | <p>Checks answer using a different sample.</p> <p>Considers whether different answers vary within an acceptable degree of accuracy.</p> |

Examples of Task Types

Plan and organise

- Find optimum solution subject to constraints.

Design and make

- Design an artefact or procedure and test it

Model and explain

- Invent, interpret, explain models.

Explore and discover

- Find relationships, make predictions

Interpret and translate

- Deduce information, move between representations of data

Evaluate and improve

- Review and improve an argument, a plan, an artefact

Plan and Organize

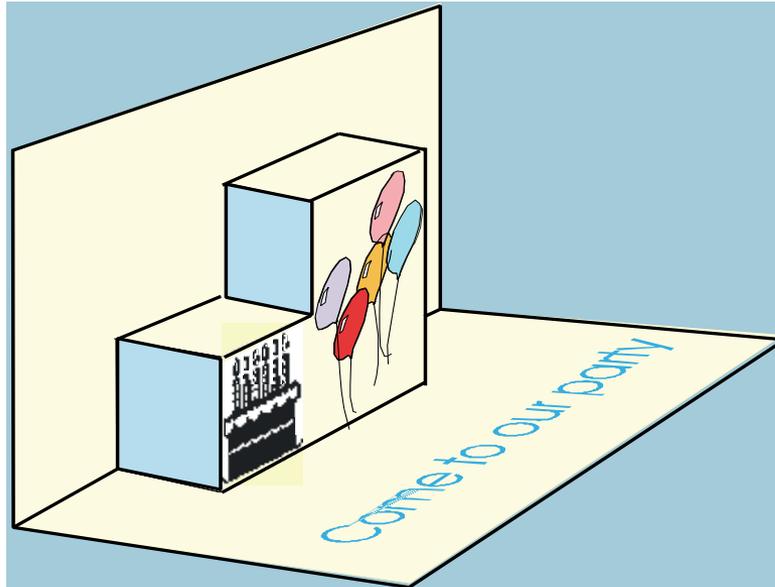
Airplane turn-round

How quickly could they do it?

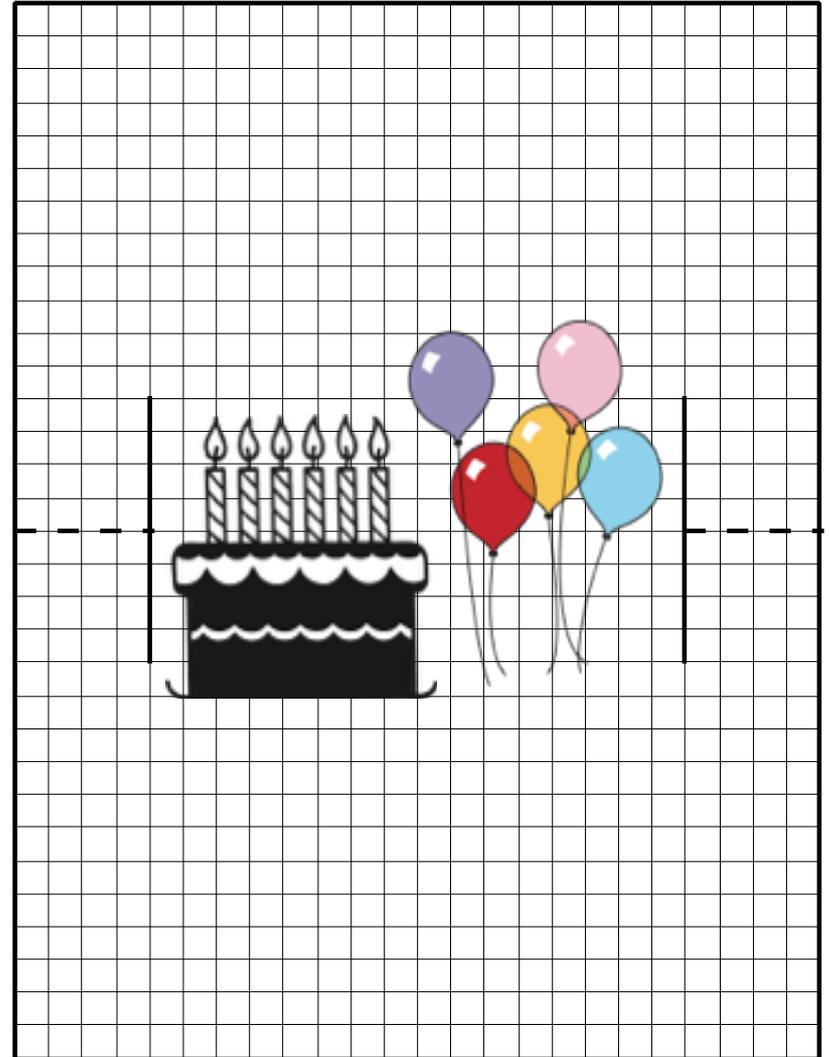


| | Job | Time needed |
|---|--|--------------------|
| A | Get passengers out of the cabin and off the plane | 10 minutes |
| B | Clean the cabin | 20 minutes |
| C | Refuel the plane | 40 minutes |
| D | Unload the baggage from the cargo hold beneath the plane | 25 minutes |
| E | Get new passengers on the plane | 25 minutes |
| F | Load the new baggage into the cargo hold | 35 minutes |
| G | Do a final safety check before take-off | 5 minutes |

Design and Make a Party Invitation



Show where you would put the cut and fold lines to make the pop-up card shown here.



Model and explain

- Babies must never be left in locked cars on hot days. They quickly dehydrate.
- Can you explain why toddlers dehydrate more rapidly than adults?

would agree, he said.

Heat kills toddler

A two-year-old girl died yesterday when her grandmother left her unattended in a car parked in a factory where she worked for seven hours. The temperature inside the car reached over 50 degrees. A police report shows that the girl's body temperature rose above 40 degrees.

Model and Explain



12 miles traffic jam on a two-lane freeway

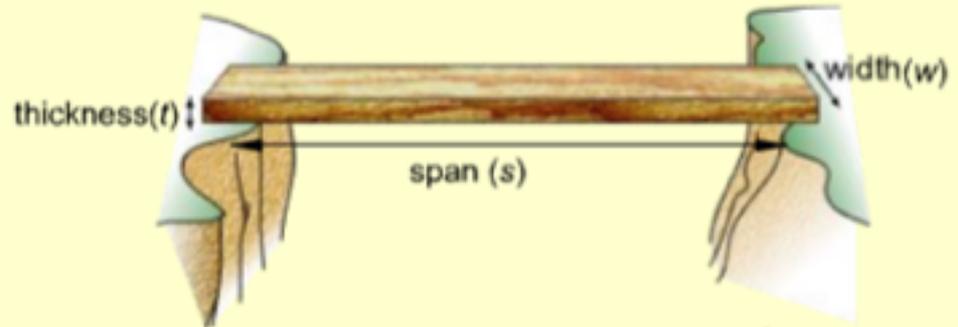
- **How many cars?**

2-second reaction time

- **How long to clear?**

Explore and discover

How does the strength of a plank bridge depend on its dimensions?



Bridge strength calculator

Width (w)

(1cm-40 cm)

?

cm

Thickness (t)

(1cm-10 cm)

?

cm

Span (s)

(1m-5 m)

?

m

Maximum weight the bridge will support

N

Explore and discover

Students control variables...

| | | | | |
|--------------|-----|-----|------|------|
| width cm | 10 | 20 | 30 | 40 |
| thickness cm | 4 | 4 | 4 | 4 |
| span m | 2 | 2 | 2 | 2 |
| weight N | 400 | 800 | 1200 | 1600 |

| | | | | |
|--------------|-----|-----|-----|------|
| width cm | 20 | 20 | 20 | 20 |
| thickness cm | 1 | 2 | 3 | 4 |
| span m | 1 | 1 | 1 | 1 |
| weight N | 100 | 400 | 900 | 1600 |

| | | | | |
|--------------|-----|-----|-----|-----|
| width cm | 30 | 30 | 30 | 30 |
| thickness cm | 2 | 2 | 2 | 2 |
| span m | 1 | 2 | 3 | 4 |
| weight N | 600 | 300 | 200 | 150 |

| | | thickness | | | | |
|-------|--------|-----------|-----|------|------|-----|
| | | 1 | 2 | 3 | 4 | |
| width | Span=1 | 10 | 50 | 200 | 450 | 800 |
| | 20 | 100 | 400 | 900 | 1600 | |
| | 30 | 150 | 600 | 1350 | 2400 | |
| | 40 | 200 | 800 | 1800 | 3200 | |

| | | thickness | | | | |
|-------|--------|-----------|-----|-----|------|-----|
| | | 1 | 2 | 3 | 4 | |
| width | Span=2 | 10 | 25 | 100 | 225 | 400 |
| | 20 | 50 | 200 | 450 | 800 | |
| | 30 | 75 | 300 | 675 | 1200 | |
| | 40 | 100 | 400 | 900 | 1600 | |

Evaluate and Improve

Here is a plan for a cycling trip to Chatsworth.

Identify the mistakes in the plan and correct them.

Meet at Loughborough station at 7.23 am. Buy tickets and then catch the train to Derby. This arrives at 7.51 am.

At Derby, catch the 8.20 am train to Cromford. This arrives at 8.41 am.

Here are the instructions for getting to the Cycle Hire centre:

“Turn left as you come out of Cromford station, walk along by the river and down Mill road. Cross over the A6. Walk up Cromford hill for about 1/2 mile and you will see

Cycle hire information

| CYCLE HIRE CENTRE | LOCATION | JAN | FEB | MAR | APR | MAY | JUN | JUL | AUG | SEPT | OCT | NOV | DEC | JAN '87 | FEB '87 | MAR '87 |
|-------------------|----------|-----|-----|-----|-----|-----|-----|-----|-----|------|-----|-----|-----|---------|---------|---------|
| MIDDLETOWN TWP | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... |
| RENFIELD | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... |

CYCLE HIRE CENTRES

WHERE TO? WHEN?

...

Average cycling speed is about 8.

Saturdays

| | | | | | | | | | | | | | | | | |
|--------------------|------|------|------|------|------|------|------|------|------|------|------|-----|-----|-----|-----|-----|
| Derby Central | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... |
| Derby North | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... |
| Derby | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... |
| London St. Pancras | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... |
| Nottingham | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... |
| Derby | 0630 | 0630 | 0643 | 1115 | 1215 | 1422 | 1523 | 1724 | 1832 | 1932 | 2012 | ... | ... | ... | ... | ... |
| Outland | 0617 | 0627 | 0654 | 1122 | 1242 | 1429 | 1530 | 1731 | 1839 | 1939 | 2019 | ... | ... | ... | ... | ... |
| Seiper | 0641 | 0631 | 0658 | 1126 | 1246 | 1433 | 1534 | 1735 | 1843 | 1943 | 2023 | ... | ... | ... | ... | ... |
| Ambergate | 0647 | 0637 | 0664 | 1132 | 1252 | 1439 | 1540 | 1741 | 1849 | 1949 | 2029 | ... | ... | ... | ... | ... |
| Whalsandwell | 0651 | 0641 | 0668 | 1136 | 1256 | 1443 | 1544 | 1745 | 1853 | 1953 | 2033 | ... | ... | ... | ... | ... |
| Cromford | 0654 | 0644 | 0671 | 1141 | 1301 | 1448 | 1549 | 1750 | 1858 | 1958 | 2038 | ... | ... | ... | ... | ... |
| Matlock Bath | 0658 | 0648 | 0675 | 1145 | 1305 | 1452 | 1553 | 1754 | 1902 | 2002 | 2042 | ... | ... | ... | ... | ... |
| Matlock | 0701 | 0651 | 0678 | 1148 | 1308 | 1455 | 1556 | 1757 | 1905 | 2005 | 2045 | ... | ... | ... | ... | ... |



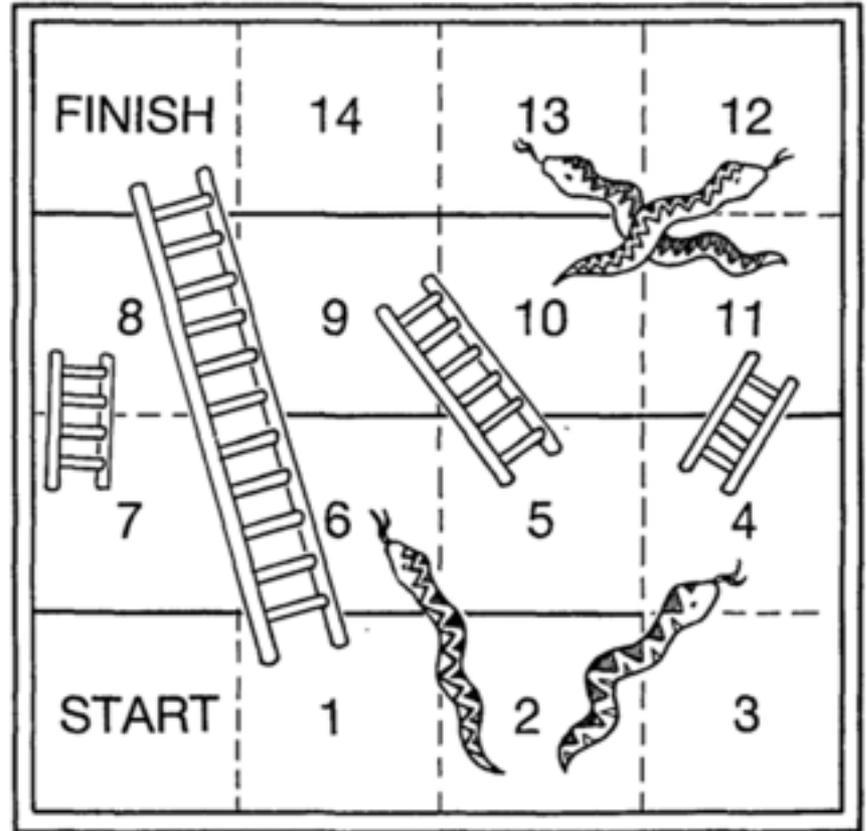
Evaluate and Improve

Snakes and Ladders Game:

Toss a coin

- Heads: move 2 spaces
- Tails: move 1 space

1. Find and describe design faults
2. Now design a better version



Problem Solving Examination Schemes (1987-95)

JMB Numeracy Certificate

- Coursework
- Basic
- Standard and Extension level certificates

NEA Syll. D Limited Grade Numeracy GCSE

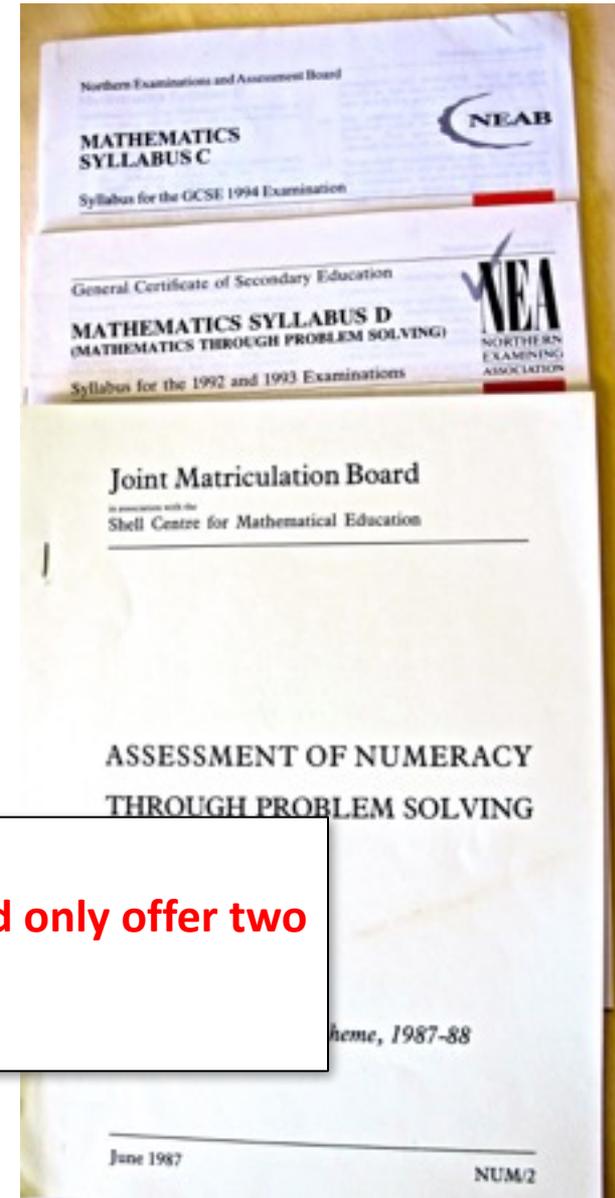
- Grades E, F, G available only.
- 50% coursework
- 25% module exams spread over two years
- 25% end of course exam.

NEAB Syll. C Full Grade Mathematics GCSE

- Grades A-G available

Then...

Government decided that every awarding body could only offer two GCSE syllabuses ... so it all disappeared!

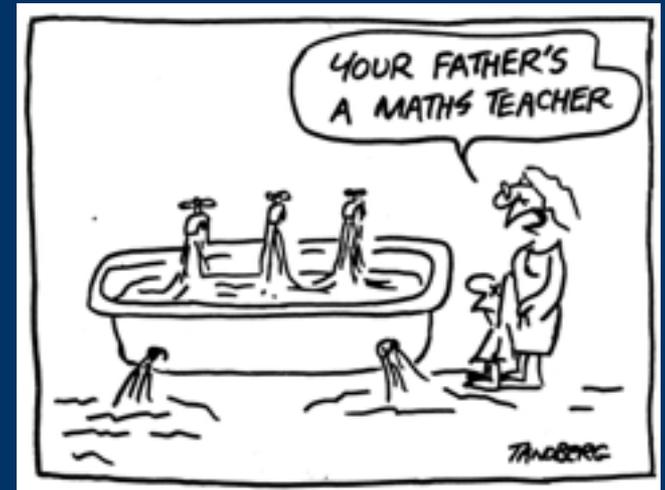


The system-level challenge

We know how to enable teachers to teach

much better mathematics
notably modelling
much more effectively

So why doesn't it happen in
classrooms?



The Systemic Problem

Policy initiatives are:

- often misguided
- usually badly designed
- outcomes far from the policy intentions

Mistakes in design lead to unintended consequences that are:

- not only predictable
- often predicted
- usually avoidable

The Big Picture

In maths, modelling is seen as important but:

- **Education systems – are complex**
- **Different levels have different pressures/priorities:**
 - Politicians – stay in power, avoid negatives
 - Superintendents, Principals – management urgencies
 - Teachers – many pressures and needs, poor support
 - Parents
- **Making a change positive for all is a challenge**
- **Politicians' world – and world view dominate**

Improvement challenges are underestimated

Potential contribution of good “engineering” ignored

Models of Change: very brief!

- No established model, but we know a lot about it
- Difficult, all key constituencies have to move
- Needs **incentives + pressure + support**
- **Alignment is vital**
- **Issues:**
 - Big bang v gradual
 - Ambition v robustness
 - Arranging piloting
 - Engines for improvement
- **Strategic design is vital**



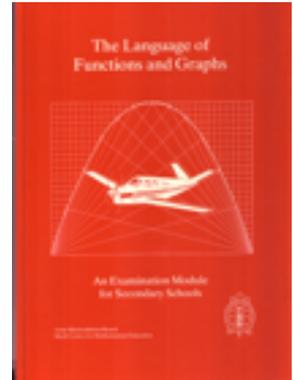
Box Model for Change

This model of change was both successful and popular with all involved.

Key features:

- **Gradual change**
 - one seriously new task-type a year in 16+ exam
~ 5% of 2-year syllabus ~ 3 weeks teaching
- **Box of support containing:**
 - 5 exemplar exam tasks, illustrating variety to be expected
 - well-engineered supportive teaching materials
 - DIY PD kit for use by teachers
- **Outcomes:**
 - Kids learned, and enjoyed the new stuff – teachers too*

* The model died, incidentally, through a re-organisation



Strategic design

“Those aspects of design that relate to the interaction of the product with the system it aims to serve”

www.educationaldesigner.org issue 3

- **Poor strategic design is the main source of:**
 - low impact
 - unintended consequences
 - Does this test assess the intended curriculum in a balance way?
 - Does it encourage good teaching?”
 - If not, point it out – and push for better tests

Strategic design: testing disasters

1. The “tests are just measurement” fallacy

- In fact they dominate teaching and learning

2. Accepting cheap limited “proxy tests”:

- Multiple choice, computer adaptive hold standards down

3. Criterion-based testing drive down standards

- They force you to test the bits separately

Good engineering methodology

- **Standard engineering:**
research > design > development > robust product
- **Design principles for these purposes**
 - Create designs, from research insights & prior designs
 - Draft designs for comment
 - Draft version for trialing (~ piloting, field testing)
 - Rich and detailed feedback from observers
 - Revision based on evidence
 - Iterate until outcomes match goals
- **Release to the field, monitor feedback**

Changing policy making

- **Not easy - work with policy makers**
- **Treat policy initiatives as serious design and engineering problems:**
 - Models of change
 - Strategic design
 - Good detailed design and engineering
- **Hasten slowly – but with short-term rewards**
- **Use design expertise**
- **Reshape the research enterprise**

Research Strategy: a design challenge for the community

How well do we, the research community, serve:

- Ourselves?
- The education system as a whole?
 - Students?
 - Teachers?
 - Support?
 - Policy makers?

How could we do (much) better?

To be a research-based field of practice:

What is needed? How do we score?

Key elements:

- A body of generally accepted knowledge
- A system for turning research insights into effective tools and processes, with rich evaluative formative feedback

Why is education not like this?

Research in Education

Three research traditions:

- **Humanities**

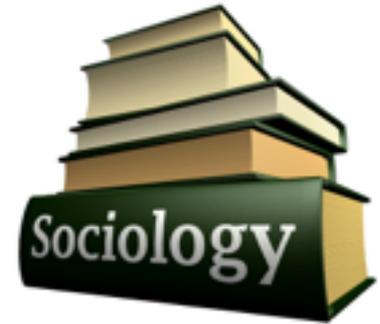
- Critical analysis > Plausible insights
- No empirical testing, so can cover a lot of ground
Treated as opinion, and politicians have opinions too.
Still, alas, has most influence on policy – but not a lot.

- **Science**

- Empirical testing of hypotheses > Insights
- Mostly very small scale > limited evidence

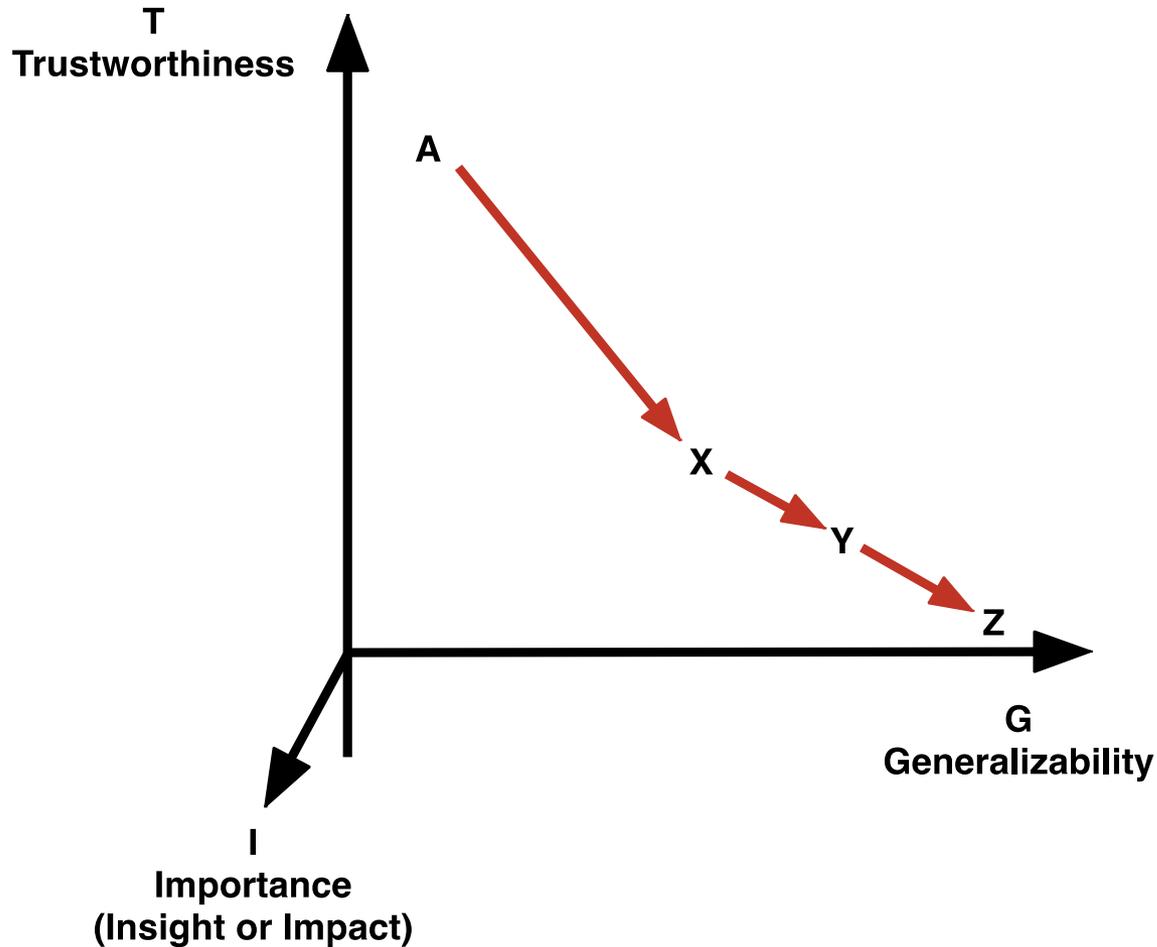
- **Engineering**

- Impact focus > Research + Design + Development
> Improved tools, processes
- Specific reproducible (= researchable) tools and treatments



3-D model of research studies

Alan Schoenfeld looked at it this way



Academic value system

Favours

| | | |
|-------------------|-------------|---------------------------|
| new ideas | <i>over</i> | reliable research |
| new results | <i>over</i> | replication and extension |
| trustworthiness | <i>over</i> | generalizability |
| small studies | <i>over</i> | major programs |
| personal research | <i>over</i> | team research |
| first author | <i>over</i> | team member |
| disputation | <i>over</i> | consensus building |
| papers | <i>over</i> | Tools and processes |

Priorities for helping practice are the reverse

How might we serve students better?

Levels of R&D:

| Study focus | Variables | Minimum scale |
|-------------------------|---------------------|---------------------|
| Learning | Student, task ... | 3 students (Piaget) |
| Teaching | ... + instruction | 1 classroom |
| Representative teachers | ...+ teacher | 5 classrooms |
| System change | ...+ school, system | 1 school system |

The scale of R&D needed gets bigger.

What do we need to handle it?

Research-based practice & policy

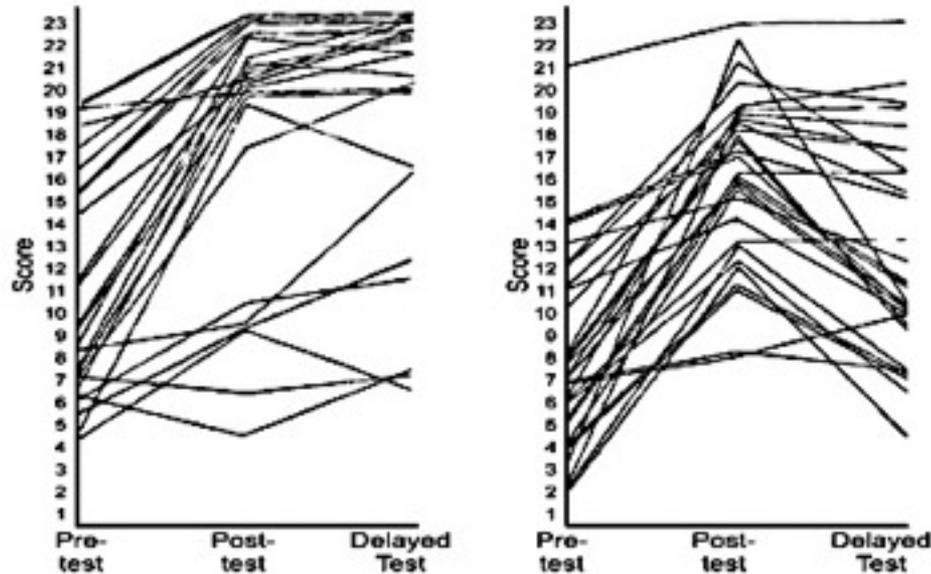
Medicine and such fields have key elements

- body of reliable research
- exemplars of tools and processes
- funded development programs
- stable design teams
- systematic iterative development
- clear circumstances of use
- comparative evaluation-in-depth

funded by clients who understand the process

Replication: exploring the boundaries

Testing models over the range of variables:



This is one result among many similar on
diagnostic teaching v direct instruction

Only common patterns lead to design principles, so we
need big programs

System Change R&D

This recognises that we face Big Problems

They need big coherent research efforts

Big collaborative R&D programs to yield

Robust solutions

Compare other fields – they are organised:

- LHC, Human Genome,**

“Big Education” can learn from “Big Science”

A practice-focused research enterprise

We should work to:

- Build collaborations for tackling big issues
- Focus on specific well-engineered exemplars
- Focus evaluation-in-depth on what happens as well as student outcomes
- Together, build bodies of reliable results with evidence of their range of validity
- Identify and publicise successes

That is “Big Education”

Math Assessment Project – a US case study

- **Strategic design** by Gates Foundation + Shell Centre
- “Common Core Standards” are challenging
- Support must be: scalable, and core of implementation
- **Formative assessment, if good, is powerful for learning**
it helps teachers and students shift roles
diagnose strengths and weaknesses,
move reasoning forward
- **Try supplementary classroom materials (10-15% of time)**
- **Who can do it, and how? Shell Centre’s 30 year record**
- **New methodology: US trials, structured observation**
- **Outcome: 5 million lesson downloads , evaluations +vex**

MathNIC a new exploratory project

Goal

- to work with 10 districts/system “partners” to design and develop ways of reconciling different level-priorities in tackling some core problems

Effective professional development

- needs to be ongoing
- adaptive expertise is challenging and takes time to build
- finding time is a challenge for the system – within the job, or outside it?
- effective support – needs R&D too

We'll see how it works!

MathNIC: how will it work?

Design is optimising within constraints

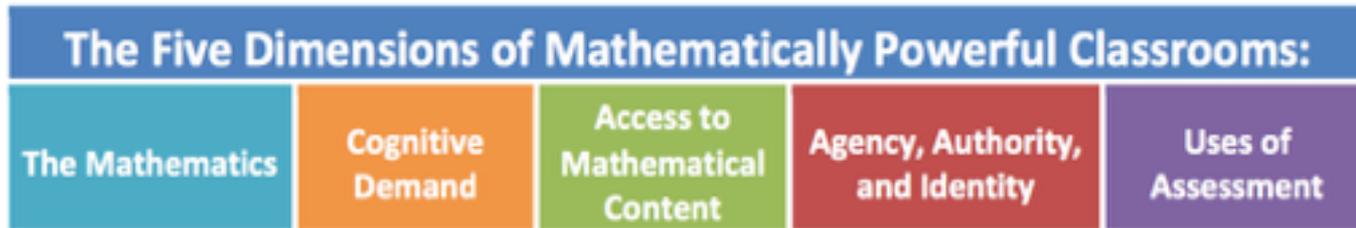
- Key challenges will be identified, then selected
- Design ideas will come from partners' experience, and the project team
- We will provide engineering: detailed design >> improvement through trialing and revision

We'll see how it works!

We have things to build on

On the MAP website you will also find, and try?

- **Professional development modules**
 - activity-based
 - design principle: plan, try, reflect together
- **TRU Math: “ Teaching for Robust Understanding”**
 - Framework for thinking about teaching: mathematics, cognitive load, access, ownership, feedback



- Tools for looking at classrooms



Thank You!